EP instabilities:
nonlinear wave-particle interactions and consequences

Lecture by M. Lesur

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Motivation

- **High-energy ions** drive **macroscopic modes** (such as AEs, EGAMs), which may lead to their **premature ejection**.

Rosenbluth, PRL (75)

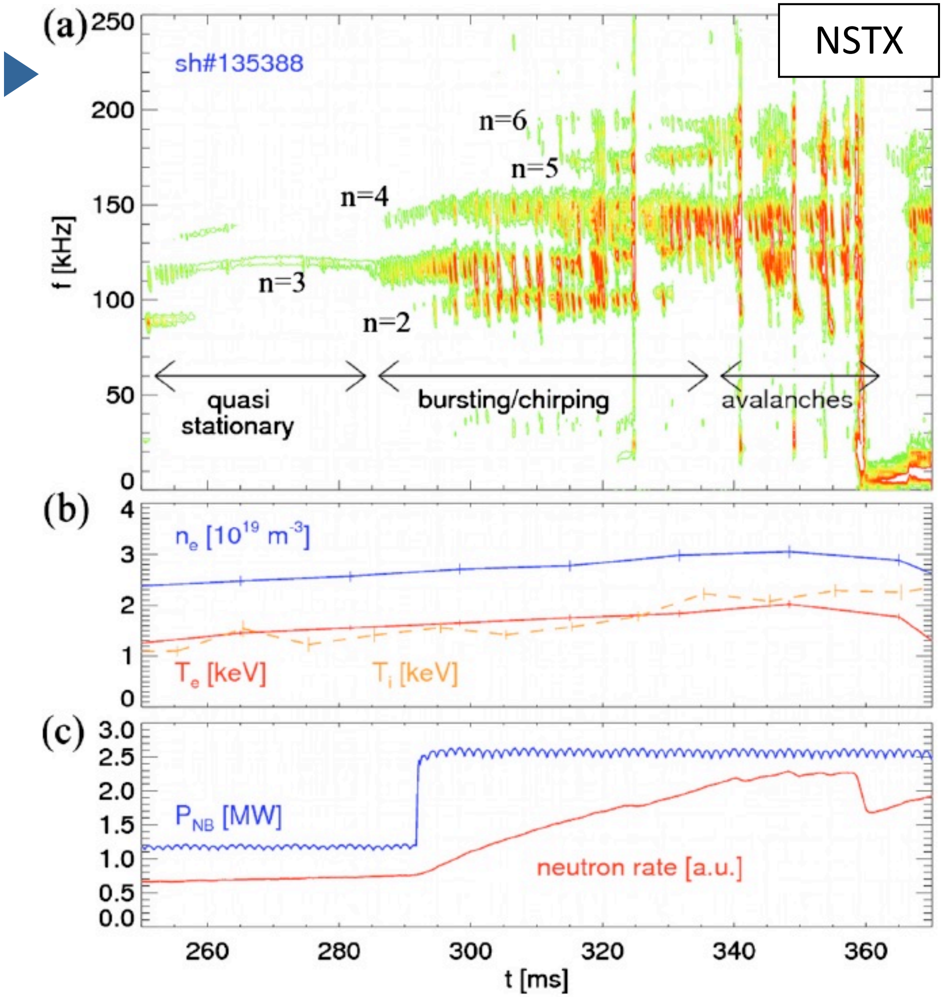
Cheng, Chen, Chance, AP (85)

- **Fast-particles loss**

Podestà, et al., PoP (10) ▶

← fluid nonlinearities (avalanches, coupling with GAM, ZF…)

← kinetic nonlinearities (particle trapping, frequency sweeping…)



- **Improving predictive capabilities**
  - stability (linear and nonlinear)
  - saturation amplitude
  - qualitative nonlinear behavior
  - coupling with other modes, with turbulence, and with flows
  - transport properties

Key points

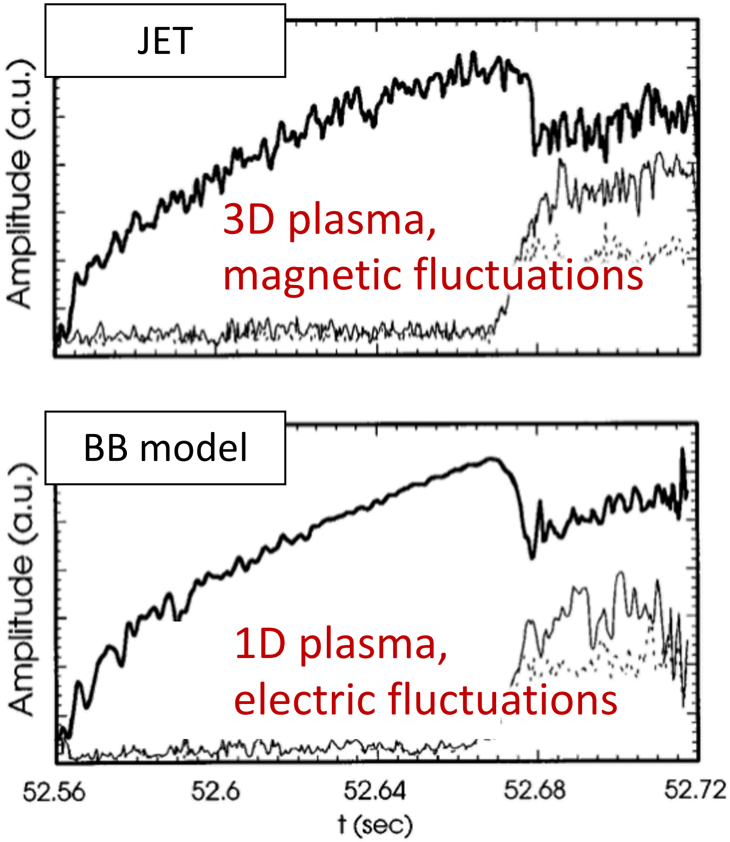
Nonlinear fast dynamics of an isolated EP-driven mode can be modeled by **the Berk-Breizman (BB) model.**

(generalization of the classic **bump-on-tail** instability)

Berk, Breizman, PFB(90)
Berk, Breizman, et al., PRL(96)
Berk, Breizman, et al., PoP(99)

- Observed quantitative similarities between BB model, and TAEs.

Fasoli, et al., PRL(98) ▶



- Rich dynamics, very informative in terms of nonlinear behavior
- Points-of-view of particles, of wave amplitude, of power balance
- Theory relates nonlinear features with linear parameters

Outline

➡ I. Bump-on-tail instability

II. The Berk-Breizman model

III. Chirping

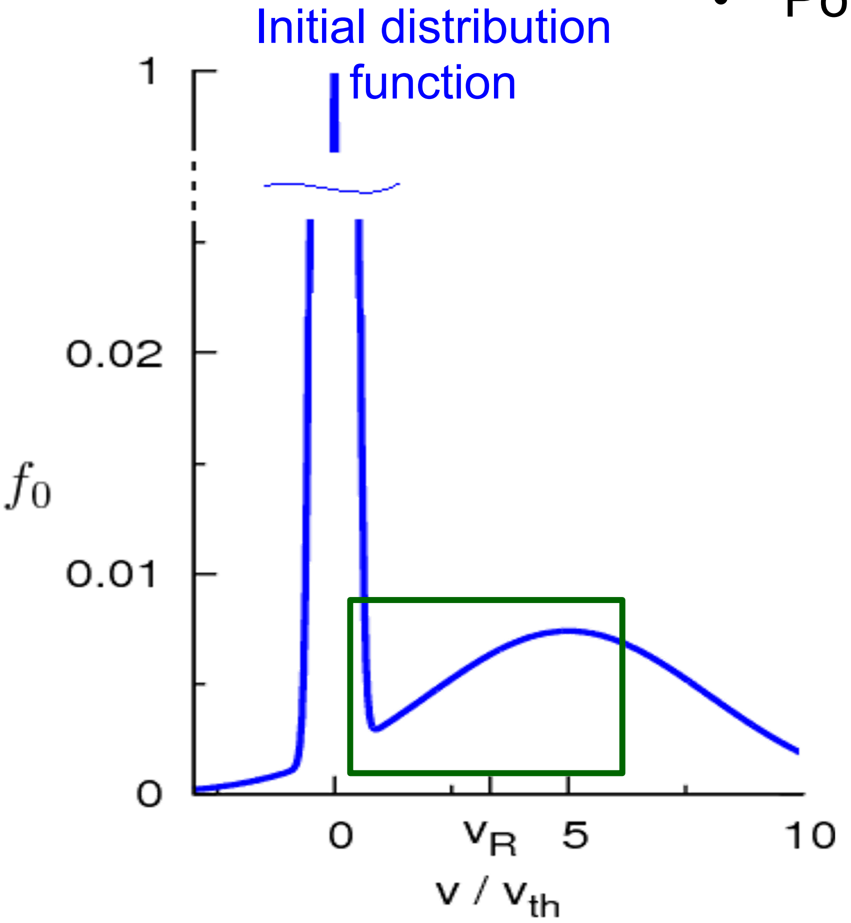
IV. Subcritical instabilities

Perspectives

1D bump-on-tail model

- 1D Vlasov equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{qE}{m}\frac{\partial f}{\partial v} = 0$$



- Poisson equation

$$\frac{\partial E}{\partial x} = \frac{q}{\epsilon_0}\int \delta f dv$$

$$\delta f = f(x,v,t) - f_0(v)$$

- Note: equivalent to "Displacement Current Equation"

$$\frac{\partial E}{\partial t} = -\frac{q}{\epsilon_0}\int v\,\delta f\,dv$$

(with Poisson at t=0)

Here: periodic B.C. in x

Instability mechanism (single mode)

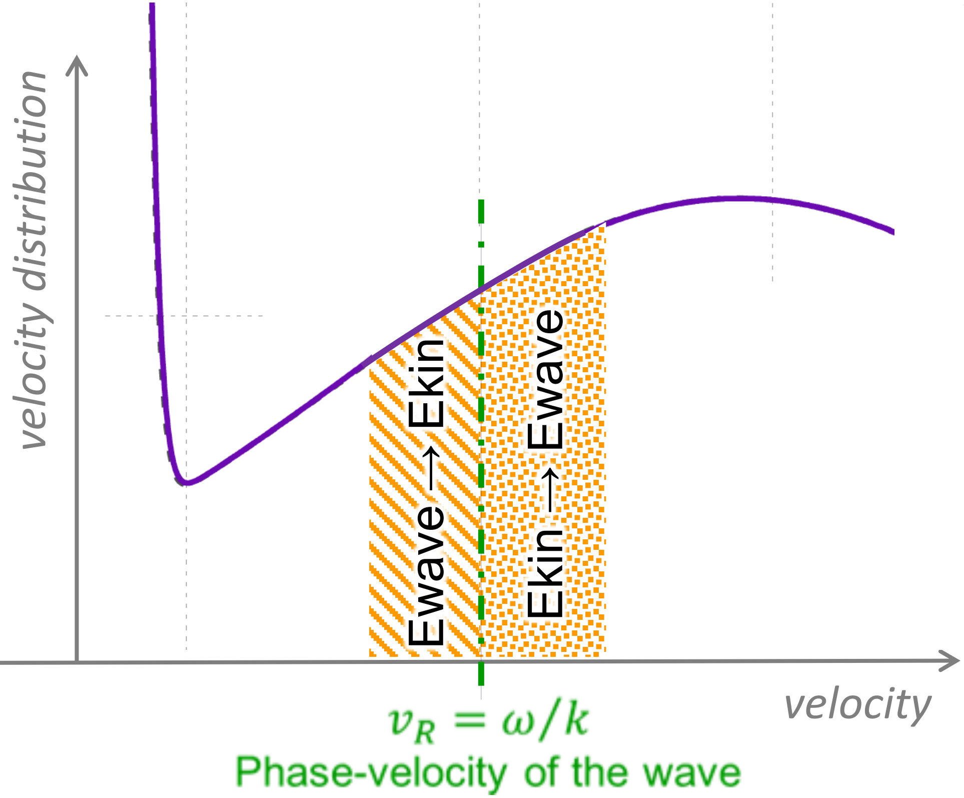
Single mode $(\boldsymbol{k}, \boldsymbol{\omega})$

$$E(x,t) = E_0(t)\, cos(kx - \omega t)$$

Resonance – assuming constant velocity, $x(t) = x_0 + v\,t$

$$\langle E(x,t)\rangle_t \to 0 \quad \text{except if} \quad v = \omega/k \equiv v_R$$

$$\hookrightarrow E(x,t) = E_0(t)\, cos(kx_0) \approx \text{const.}$$



Near resonance → synchronisation

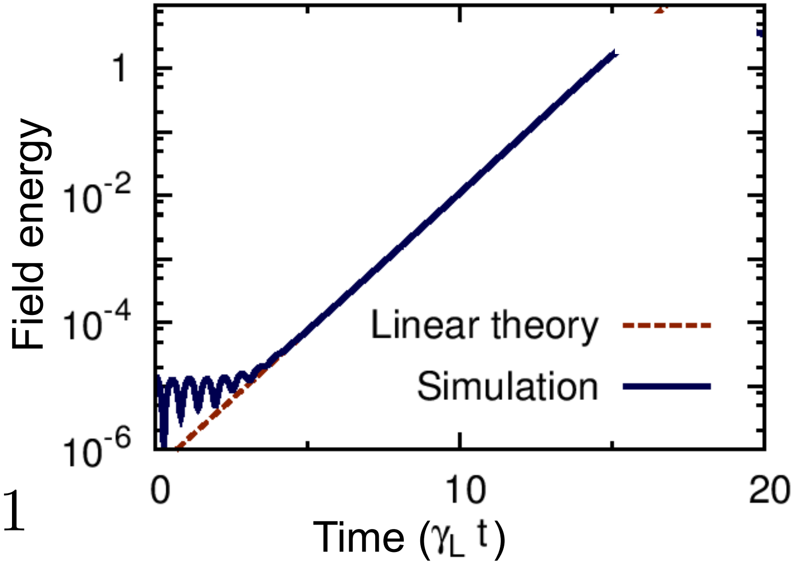
(velocity is not constant)

Positive slope of velocity distribution at the resonant velocity

↳ on average, Ekin → Ewave

Linear theory

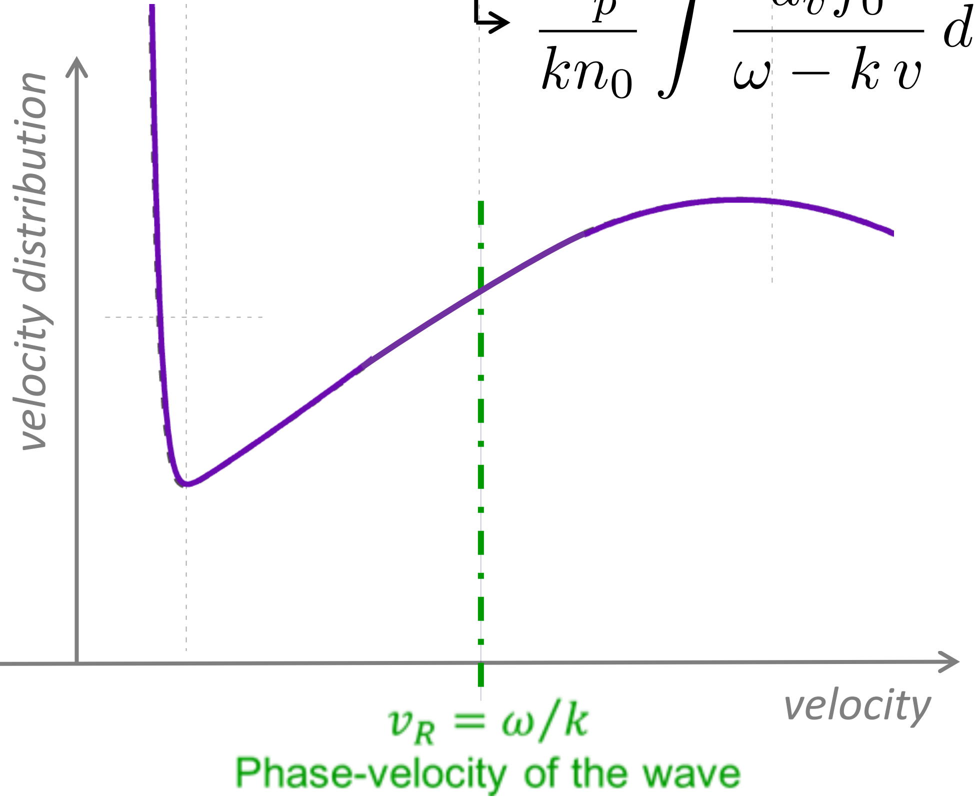
Linear theory predicts exponential growth



$$f(x,v,t) = f_0(v) + \delta f \qquad \delta f << f_0$$

↳ $$\frac{\partial \delta f}{\partial t} + v\frac{\partial \delta f}{\partial x} + \frac{qE}{m}\frac{\partial f_0}{\partial v} = 0$$

↳ $$\frac{\omega_p^2}{k n_0}\int \frac{d_v f_0}{\omega - k\,v}\,dv \;=\; 1$$



↳ $$E_0(t) \sim e^{\gamma_L t}$$

with linear growth rate,

$$\gamma_L \sim \frac{\omega^3}{k^2}\;\frac{df_0}{dv}\Big|_{v=\omega/k}$$

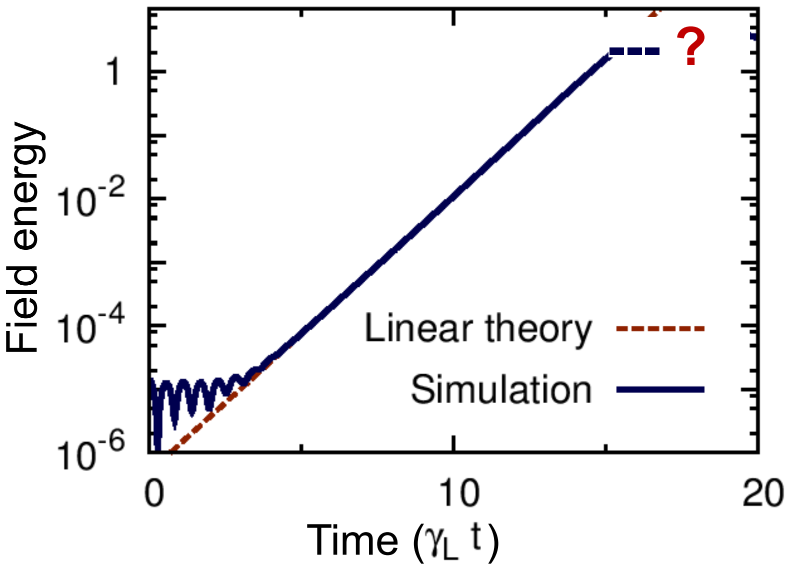
⚠ $\gamma \ll \omega$

$\omega/k \gg v_{\rm th}$

Introducing kinetic nonlinearity

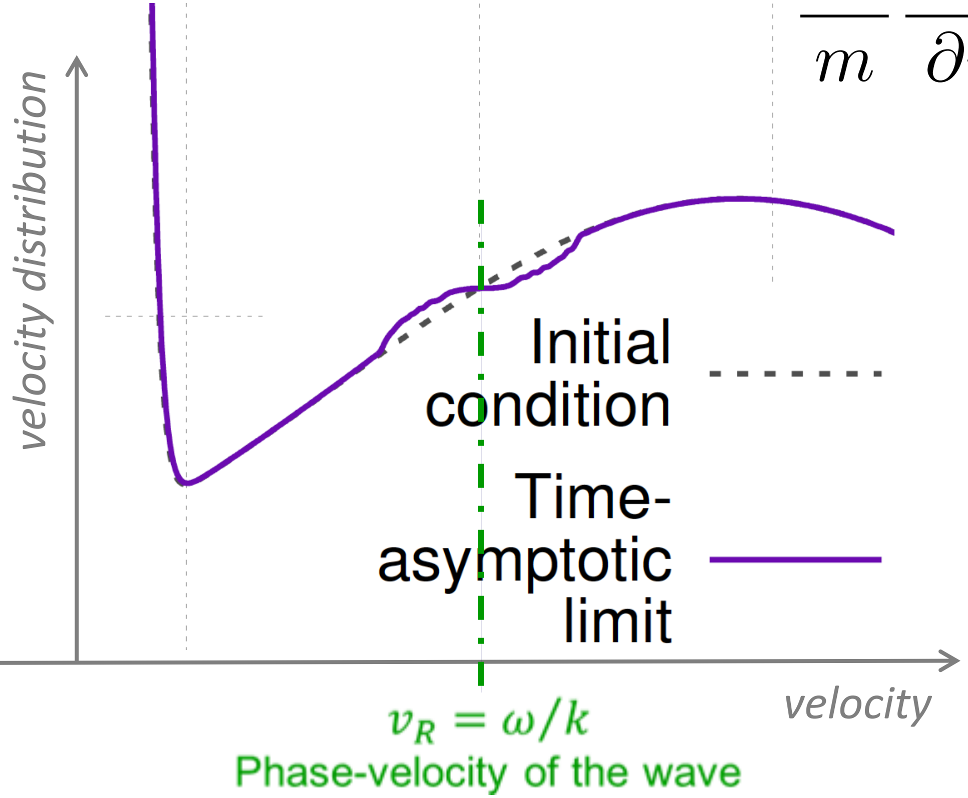
Linearly, $E_0(t) \sim e^{\gamma_L t}$ $|\delta f| \sim e^{\gamma_L t}$

$$\left|\frac{\partial \delta f}{\partial v}\right| \sim e^{\gamma_L t}$$



⇒ At some point, need accounting for

$$\frac{qE}{m}\frac{\partial \delta f}{\partial v}$$



⇔ accounting for nonlinear motion

$$\cancel{x(t) = x_0 + v\,t}$$

$$x(t) = x_0 + \int_0^t v(t')\,dt'$$

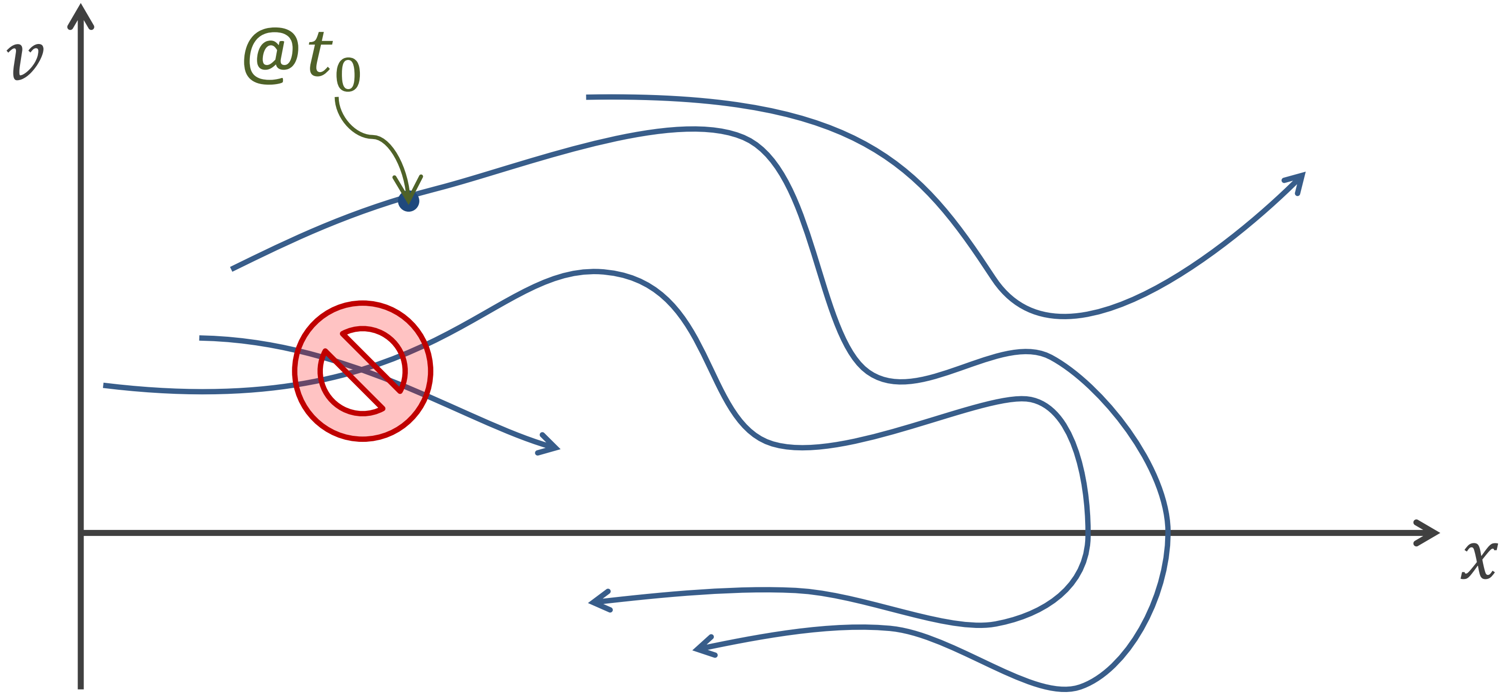
$$v(t') = v_0 + \frac{q}{m}\int_0^{t'} E[x(t''), t'']\,dt''$$

⇒ Let’s go into phase-space

1D motion → 2D phase-space

Particle orbits live in the 2D phase-space (x,v)

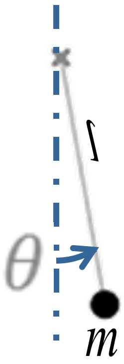
$$\frac{d}{dt}\binom{x}{v} = \binom{v}{qE(x,t)/m}$$



Vlasov ⟺ $$\frac{\partial f}{\partial t} + \frac{dx}{dt}\frac{\partial f}{\partial x} + \frac{dv}{dt}\frac{\partial f}{\partial v} = \left.\frac{df}{dt}\right|_{\mathrm{traj.}} = 0$$

⇒ Distribution function is constant along phase-space ~~orbits~~ motion

Resonance

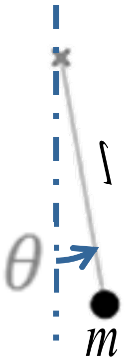


Energy of the single pendulum: $E = \frac{1}{2} m l^2 \dot{\theta}^2 - mgl\cos\theta = const.$

Energy of a charged particle in electric field: $E = \frac{1}{2} m v^2 - q\varphi_0 \cos kx$

(in the ref. frame of the wave, $v_R = \omega/k$)

Resonance



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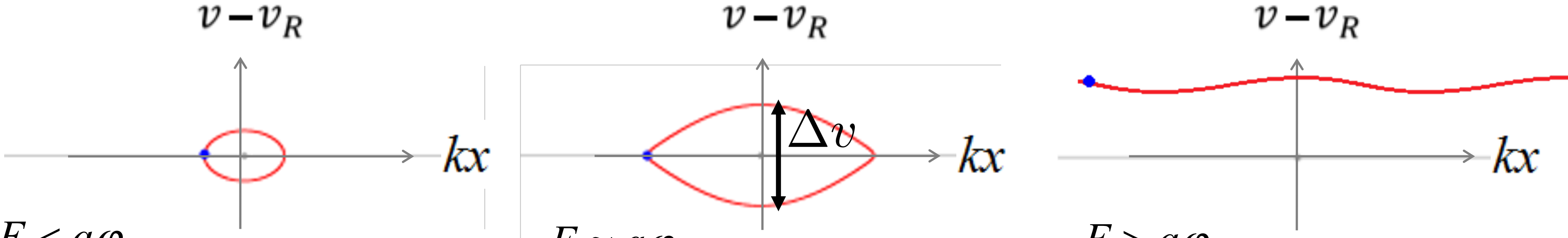


$E < q\varphi_0$ $E \approx q\varphi_0$ $E > q\varphi_0$

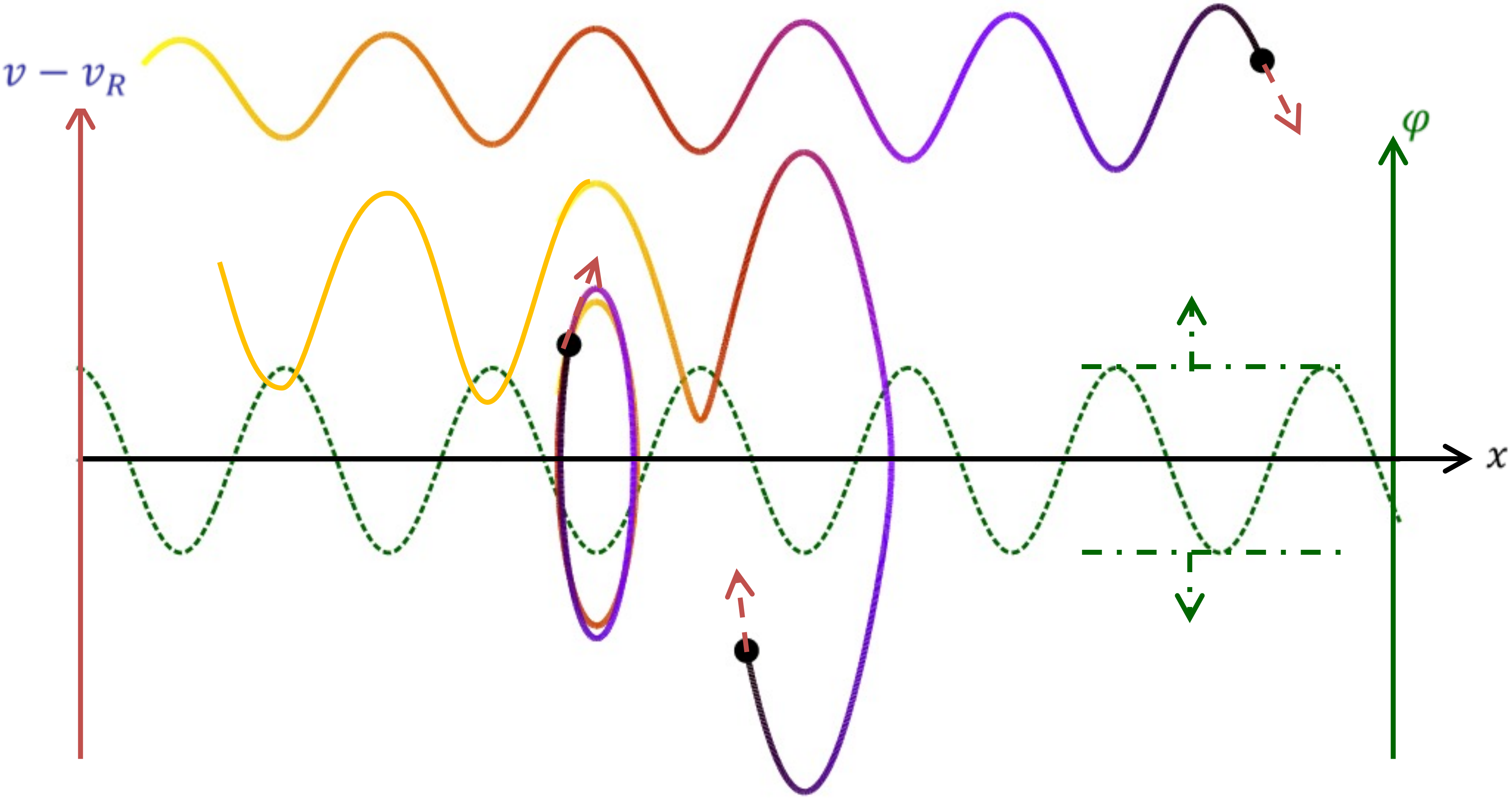
Bounce frequency of most deeply trapped particles

$$\omega_b \sim \sqrt{kE_0}$$

Island width

$$\Delta v \sim \sqrt{E_0 / k}$$

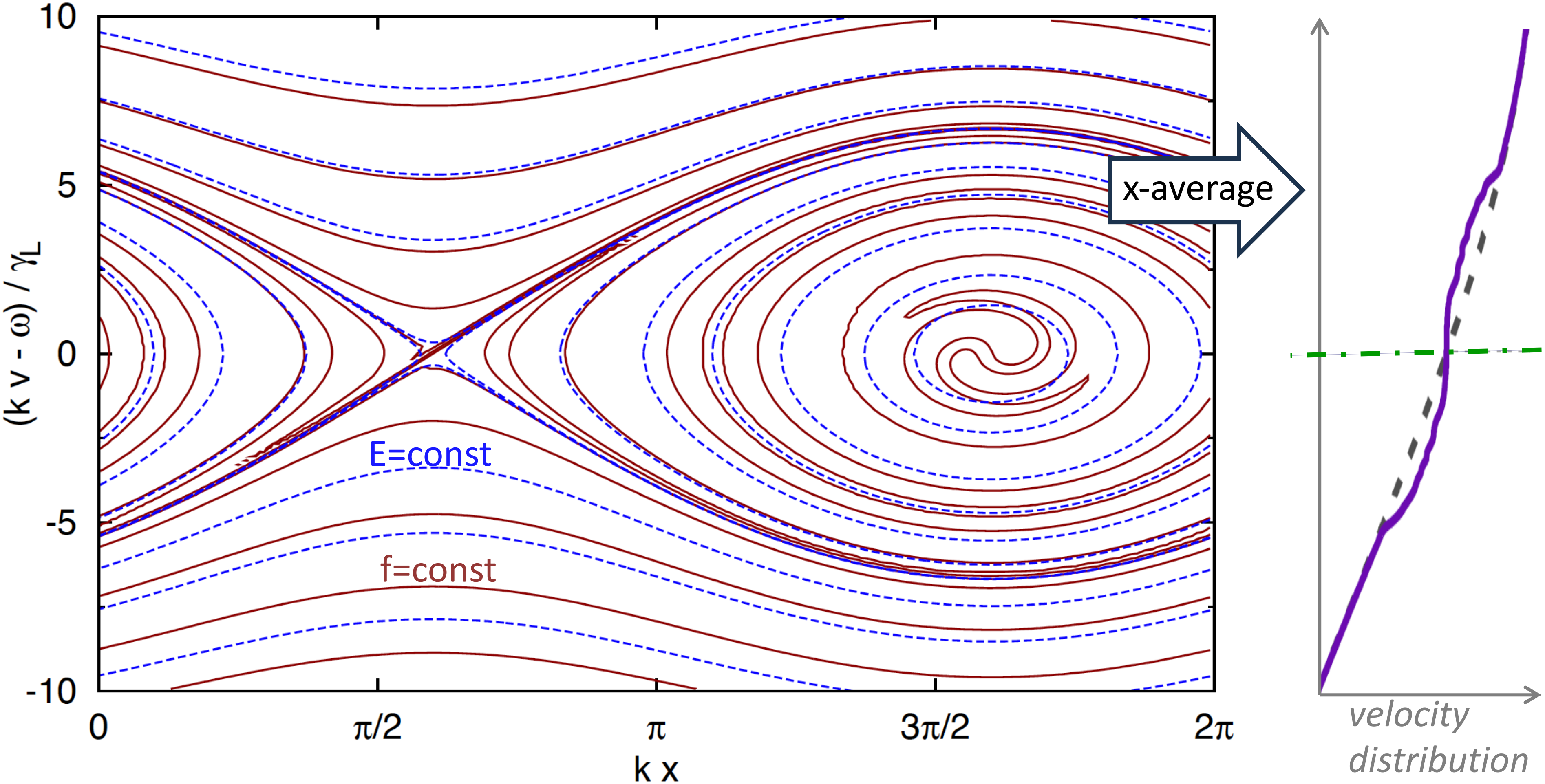
Electrostatic trapping



⇒ **As the wave amplitude grows, more and more particles are trapped**

Wave energy $\mathcal{E} \sim E^2$ Free energy $\mathcal{E} \sim E^{3/2}$

Saturation of the single mode BoT instability



Results in BGK solution: *f(E)* steady-state solution of Vlasov-Poisson

Bernstein, Greene, Kruskal '57

(assuming infinitesimal P-S diffusion)

Numerical simulations → $\boxed{\frac{\omega_b}{\gamma_L} \approx 3.2}$ (in the ideal situation – but in general sensitive to f_0)

O'Neil '71

Nonlinear theory for a single mode

Solving Vlasov equation in the resonant region…

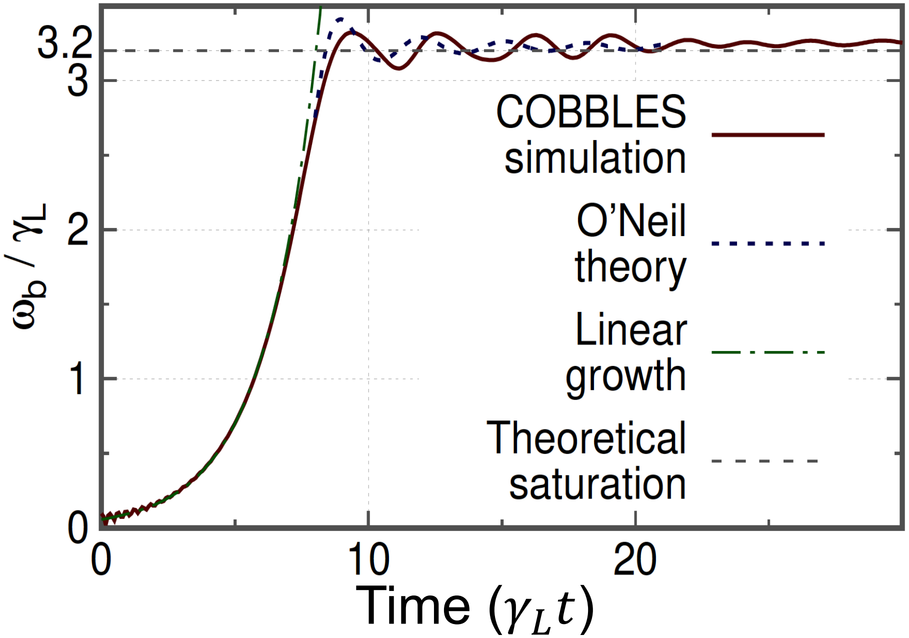
$$\frac{dT}{dt} = \gamma_L \frac{\mathcal{E}^2}{4\pi}\left\{\frac{64}{\pi}\int_0^1 \frac{d\kappa}{\kappa^3}\int_0^{\frac{1}{2}\pi}\frac{d\xi}{\pi}\,\mathrm{sn}\,(t/\kappa\tau,\kappa)\;\mathrm{cn}\,(t/\kappa\tau,\kappa)[1-\kappa^2\sin^2(\xi)]^{\frac{1}{2}}\,\frac{[2\sin^2(\xi)-1-\kappa^2\,\mathrm{sn}^2\,(t/\kappa\tau,\kappa)\sin^2(\xi)]}{[1-\kappa^2\,\mathrm{sn}^2\,(t/\kappa\tau,\kappa)\sin^2(\xi)]^2}\right.$$
$$\left.+\frac{64}{\pi}\int_1^\infty \frac{d\kappa}{\kappa^5}\int_0^{\pi/2}\frac{d\zeta}{\pi}\,\frac{\mathrm{sn}\,(t/\tau,1/\kappa)\,\mathrm{dn}\,(t/\tau,1/\kappa)\cos^2(\zeta)[(2/\kappa^2)\sin^2(\zeta)-1-(1/\kappa^2)\,\mathrm{sn}^2\,(t/\tau,1/\kappa)\sin^2(\zeta)]}{[1-(1/\kappa^2)\sin^2(\zeta)]^{\frac{1}{2}}[1-(1/\kappa^2)\,\mathrm{sn}^2\,(t/\tau,1/\kappa)\sin^2(\zeta)]^2}\right\}, \quad (29)$$

O'Neil '65

$$\hookrightarrow \quad \frac{\omega_b(t)}{\omega_b(0)} = \exp\frac{\gamma_L}{\pi\omega_b}\int_0^1 \mathrm{d}\kappa\left(1-\cos\frac{2\omega_b t}{\kappa}\right)$$

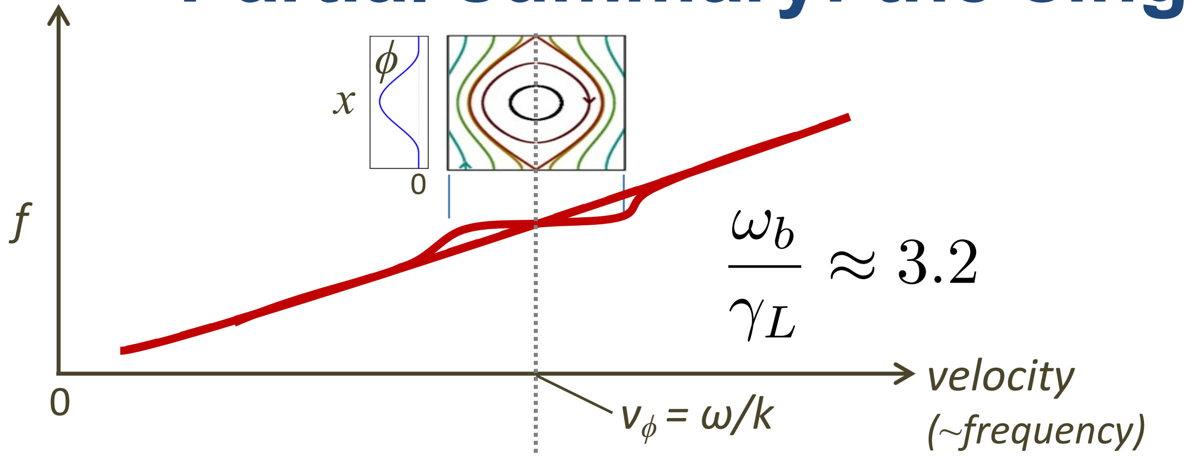
$$\omega_b \sim \sqrt{kE_0}$$

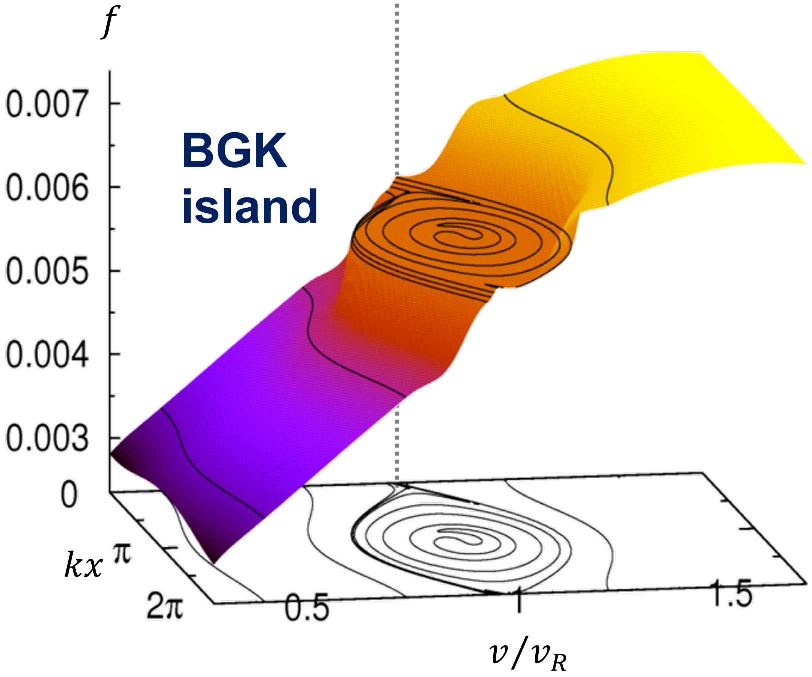
$$\gamma_L/\omega_b \ll 1$$
$$\omega/\omega_b \gg 1$$
$$\omega_b t \ll 1$$



⇒ Nonlinear theory predicts amplitude oscillations

Partial summary: the single-mode BoT





Bill quizz

In the ideal steady-state shown here, the relationship between electric field amplitude E_0 and the initial velocity slope $\partial_v f_0$ is

1. $E_0 \sim (\partial_v f_0)^{1/2}$
2. $E_0 \sim \partial_v f_0$
3. $E_0 \sim (\partial_v f_0)^2$

⇒ Island in phase-space (x,v), consistent with a finite amplitude potential

Outline

I. Bump-on-tail instability

➡ II. The Berk-Breizman model

III. Chirping

IV. Subcritical instabilities

Perspectives

The Berk-Breizman model

Classic “bump-on-tail” instability, with collisions and external damping.

Berk, Breizman, et al. ***’90****, ’92, ’93, ’95,* ***’96****,* ***’97****, ’98, ’99*

- 1D kinetic equation with collision operator,

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{qE}{m}\frac{\partial f}{\partial v} = C(f - f_0)$$

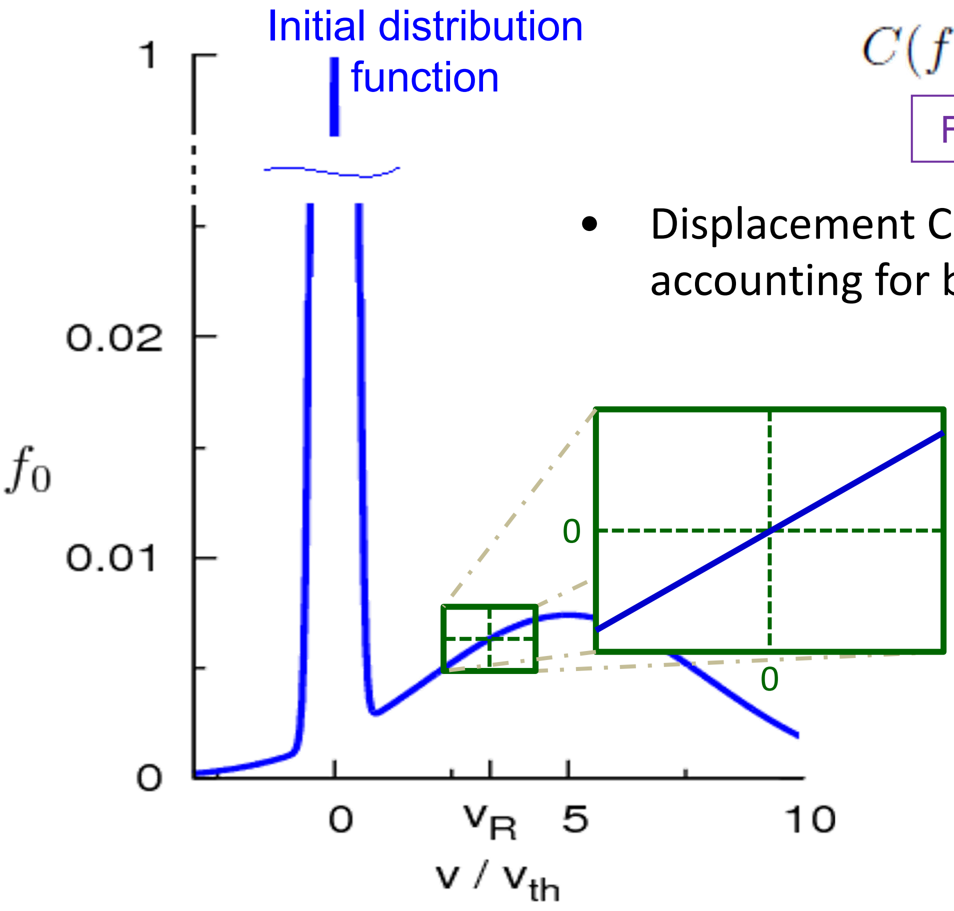
$$C(f - f_0) = -\nu_a\,(f - f_0)$$

Krook

or

$$C(f - f_0) = \underbrace{\frac{\nu_f^2}{k}\frac{\partial\,(f - f_0)}{\partial v}}_{\text{friction (drag)}} + \underbrace{\frac{\nu_d^3}{k^2}\frac{\partial^2\,(f - f_0)}{\partial v^2}}_{\text{v-space diffusion}}$$

F-P



- Displacement Current Equation with an external wave damping accounting for background dissipative mechanisms,

$$\frac{\partial E}{\partial t} = -4\pi q\int v(f - \bar{f})\mathrm{d}v \boxed{-2\gamma_d E}$$

x-averaged

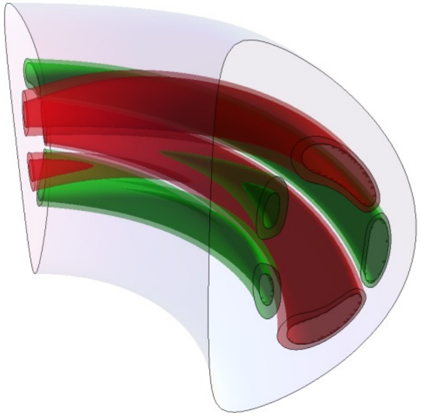
- Single electrostatic wave,

$$E(x,t) = E_0(t)\cos(kx - \omega t)$$

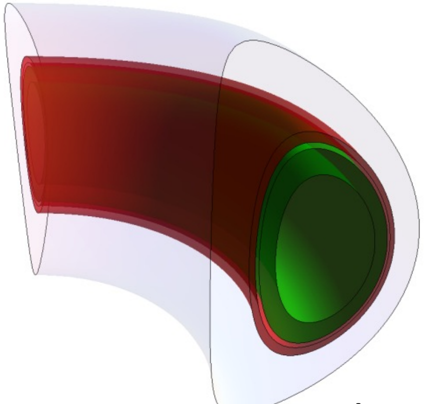
fixed

- Reduction to near-resonance perturbative model

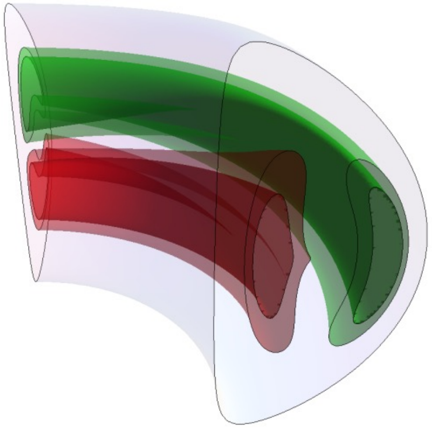
Nonlinear dynamics is essentially 1D



- Alfvén waves



- Geodesic Acoustic Modes (GAMs)



- Fishbones

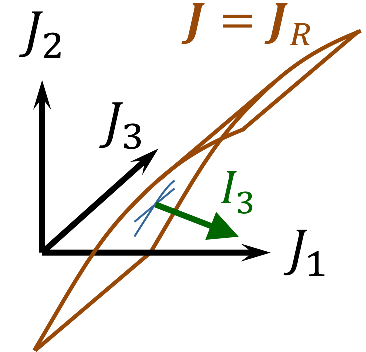
Complementary approaches

$$H_1(\boldsymbol{\alpha}, \boldsymbol{J}, t) = V(\boldsymbol{J}, t)\sin(\boldsymbol{n}\cdot\boldsymbol{\alpha} - \omega t)$$

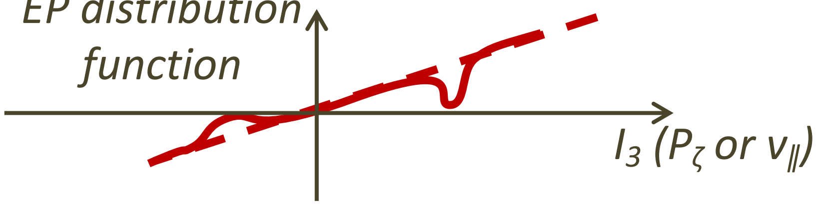
$$\boxed{\begin{array}{c}\theta_3 = \boldsymbol{n}\cdot\boldsymbol{\alpha} - \omega t \\ I_3 \sim P_\varphi - P_\varphi^{res}\end{array}}$$

Lichtenberg '69

$$H_1(\theta_3, I_3, t) = \frac{1}{2}\, D\, {I_3}^2 + V(t)\sin(\theta_3) + o(I_3, \partial_{\boldsymbol{J}} V)$$

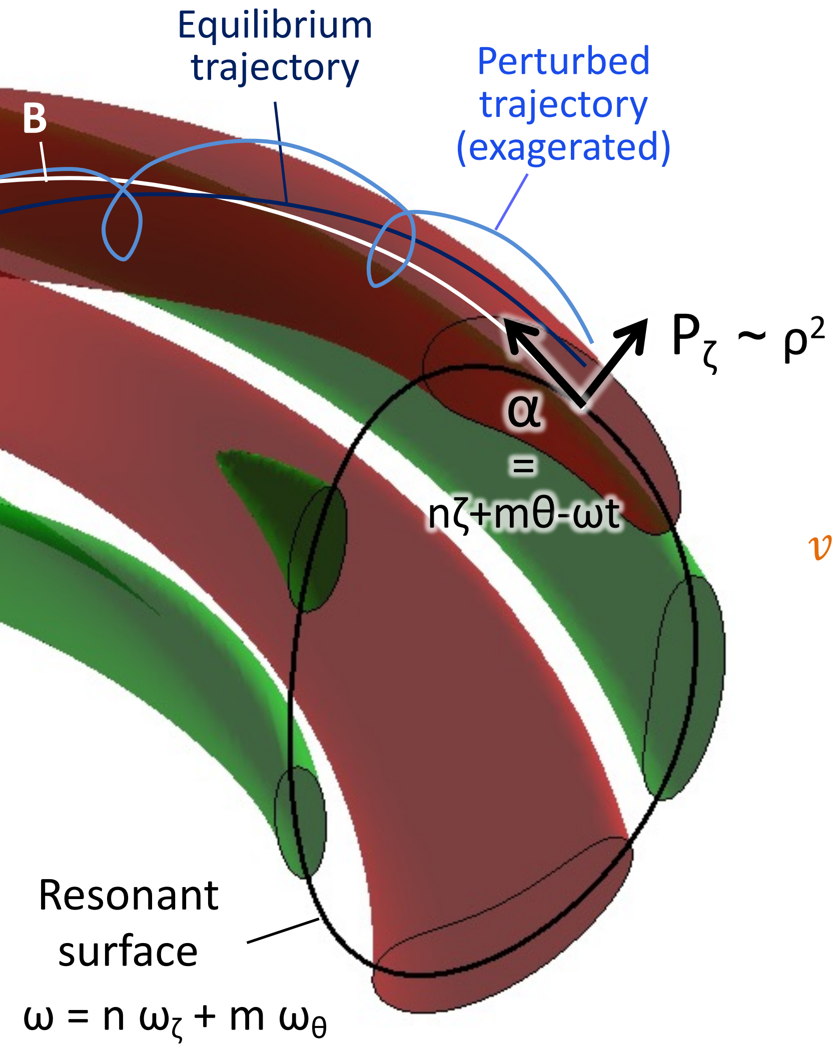


Resonance sheet in phase-space



⇒ Perturbative description of nonlinear dynamics for single, isolated mode, with fixed mode structure

e.g. Toroidal Alfvén Eigenmode (TAE)

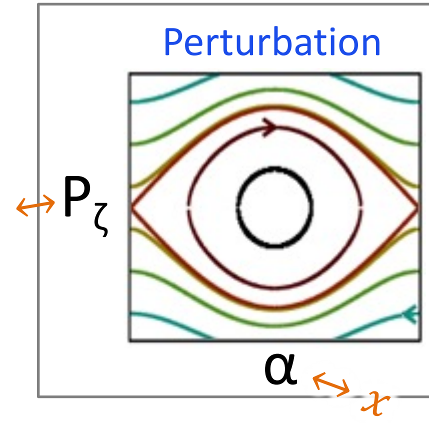


3D linear theory or experiment

⇩

Spacial mode structure, frequency, effective collision frequencies

⇩ *Input parameters*



1D model (2D phase-space)

$$V \sim \delta \boldsymbol{A}_\perp \leftrightarrow \phi$$

Adding energy dissipation

⇒ Berk-Breizman model

⇩

Nonlinear dynamics

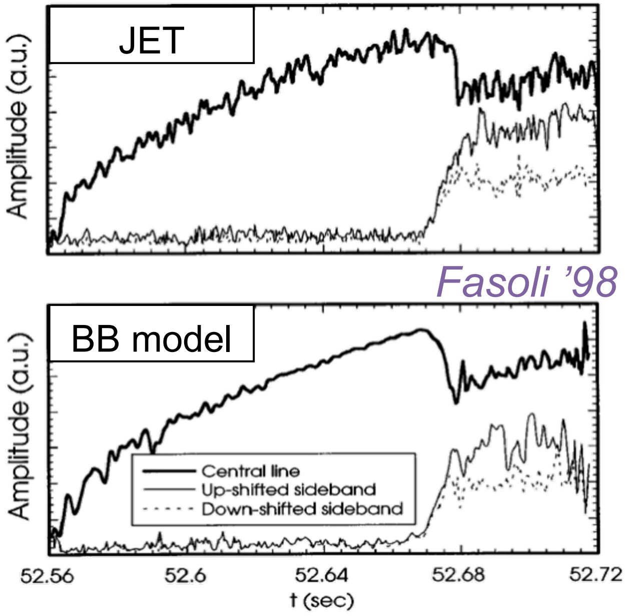
⇒ Problem split between slowly-evolving (~ 100 ms) 3D mode structure and fast (~ 1 ms) 1D nonlinear amplitude and phase dynamics

The BB model reproduces experiments

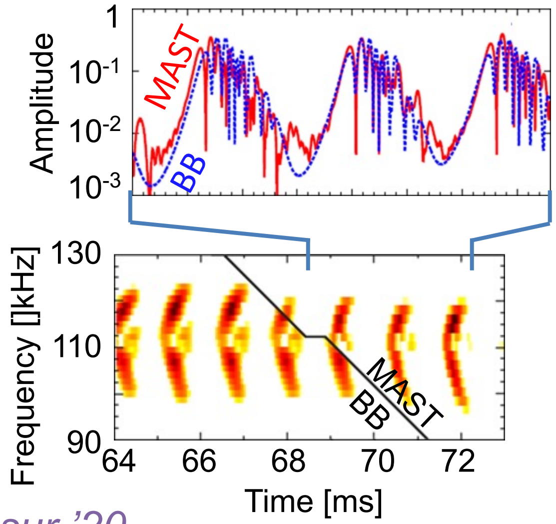
Comparing BB model with fusion plasma experiments

- **Quantitative agreement** (after adjusting free parameters)
  - TAEs on JET, JT-60U, MAST
  - EGAM on the LHD
- Qualitative agreement (may be quantitative ?)
  - TAEs on the LHD
  - EGAM on JET
  - e-fishbones on HL-2a
  - many more
- Predictions
  - Qualitative nonlinear behavior
  - EGAM on LHD: phase relationship and its evolution, amplitude threshold
  - more?

⇒ Successful reduced modeling
(albeit maybe limited predictive capabilities)



Fasoli '98



Lesur '20

Time-scales in the BB model

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{qE}{m}\frac{\partial f}{\partial v} = C(f - f_0)$$

$$C(f - f_0) = -\nu_a\,(f - f_0)$$

or

$$C(f - f_0) = \frac{\nu_f^2}{k}\frac{\partial\,(f - f_0)}{\partial v} + \frac{\nu_d^3}{k^2}\frac{\partial^2\,(f - f_0)}{\partial v^2}$$

$$\frac{\partial E}{\partial t} = -4\pi q\int v(f - \bar{f})\mathrm{d}v - 2\gamma_d E$$

Involves many time-scales:

ω	mode frequency	
ω_b	bounce frequency	$\omega_b \sim \sqrt{kE_0}$
γ_L	linear drive	≠ linear growth rate $\gamma \approx \gamma_L - \gamma_d$
γ_d	external damping rate	
ν_f	drag rate	or Krook collision rate ν_a
ν_d	diffusion rate	

$\Rightarrow$ **Rich phenomenology**

Power balance in the BB model

Wave energy $\mathcal{E}(t) \equiv \int E^2/(2\epsilon_0)\mathrm{d}x$

Power transferred power from field to particles

$$P_h(t) \equiv q\int vEf\mathrm{d}x\mathrm{d}v$$

Electric power: $\frac{\partial W}{\partial t} = \mathbf{F_E} \cdot \mathbf{v}$

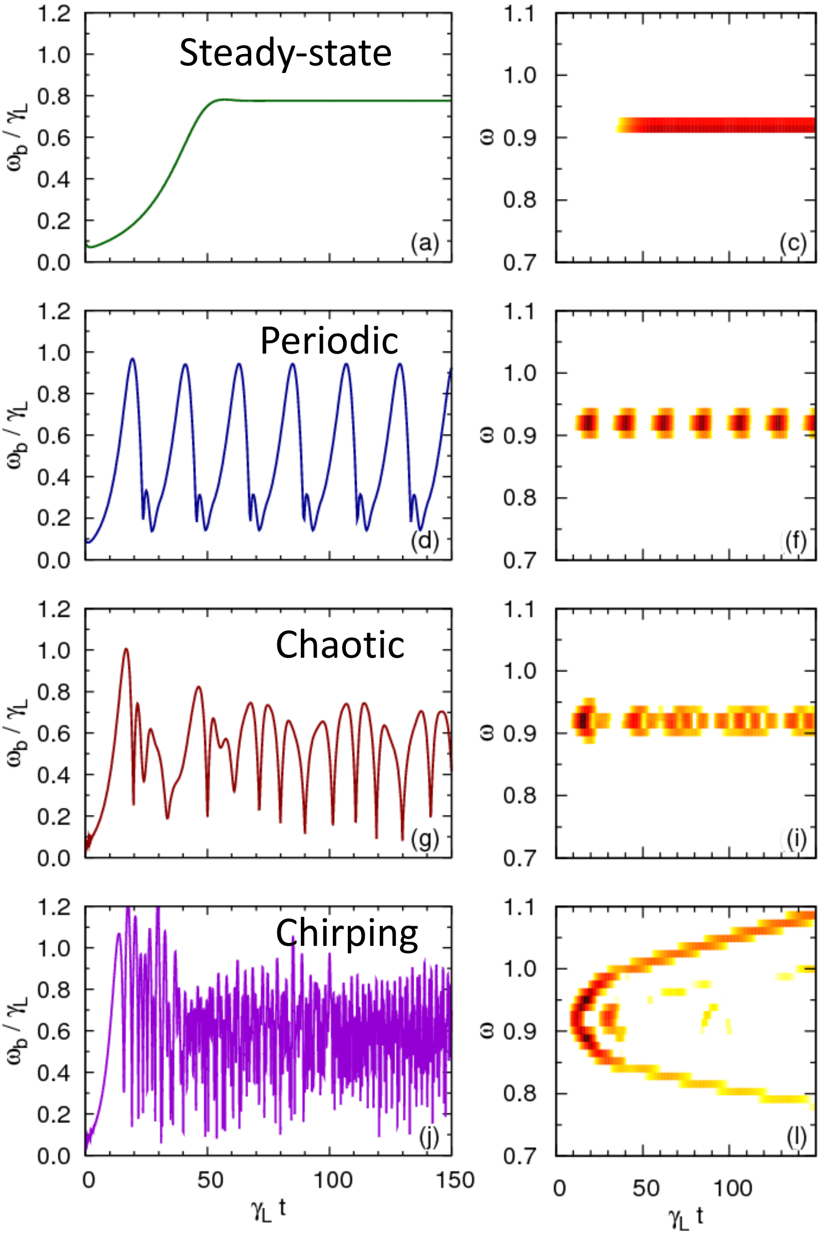
Density of electric power: $\boldsymbol{J} \cdot \boldsymbol{E}$

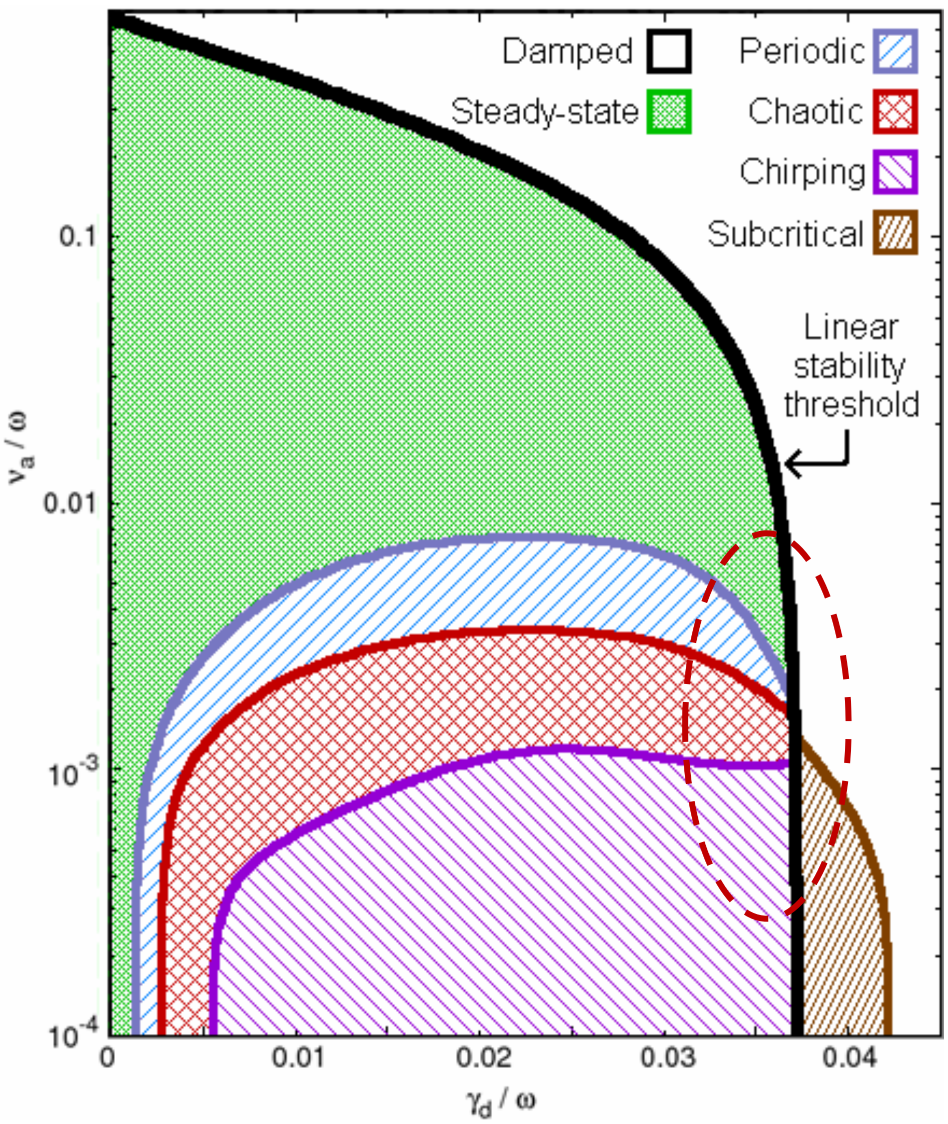
Power balance

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} + P_h + \underbrace{4\gamma_d\mathcal{E}}_{\text{dissipated power}} = 0$$

⇒ **If wave/particles power transfer can be calculated, so can be wave energy**

Phenomenology





Collision frequency (Krook)

External damping rate

Nonlinear saturation far above threshold

Regime $\gamma_d \sim \nu_a \ll \gamma_L$

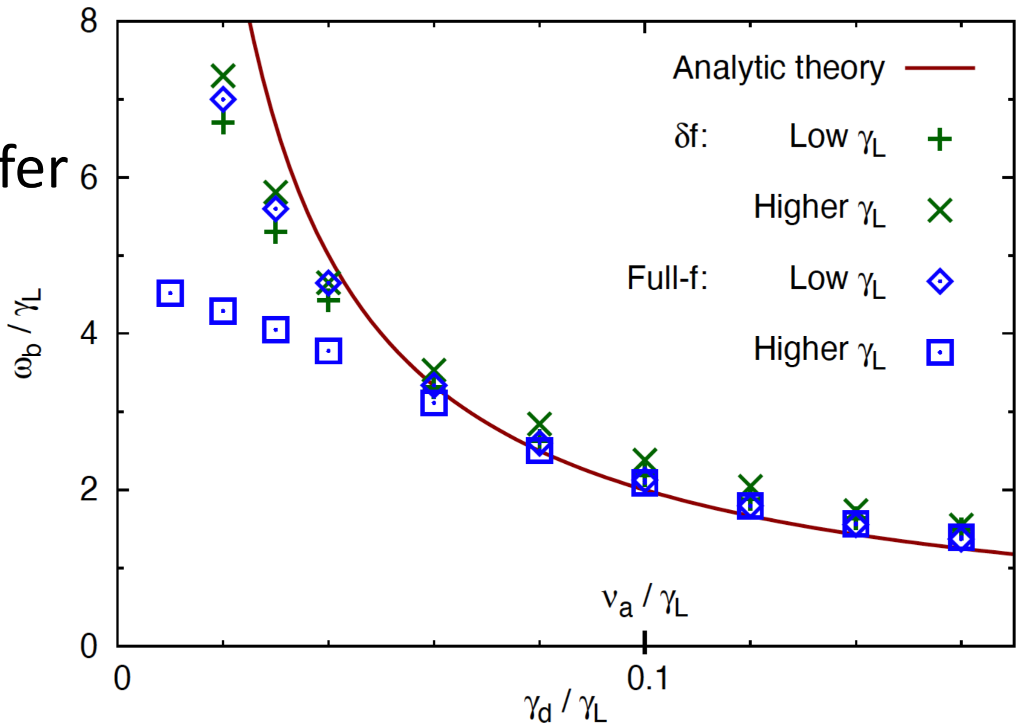
Assuming resonant power transfer dominated by a narrow region around $v = v_R$

$$4\,\omega_b/k \ll \omega/k$$

Assuming steady-state, and $\nu_a \ll \omega_b$, power balance yields

$$\boxed{\frac{\omega_b}{\gamma_L} = 1.96\,\frac{\nu_a}{\gamma_d}}$$

Berk, Breizman '90



(compare with the classic bump-on-tail case, $\frac{\omega_b}{\gamma_L} \approx 3.2$)

$\Rightarrow$ Similar (and even higher) amplitudes can be reached despite dissipation

Cubic nonlinearity near threshold

Expansion in Fourier series $f = \langle f \rangle + \sum_{n=1}^{\infty} f_n \, e^{in(kx-\omega t+\alpha)}$

Near-threshold ordering

Berk, Breizman, Pekker '96

$$\gamma \approx \gamma_L - \gamma_d \ll \gamma_L$$

$f_2 \ll f_1 \ll f_0$ and $\langle f \rangle \approx f_0$ (and $f_{n\geq 3} = 0$)

Substitute into the kinetic equation

$$\begin{aligned} \partial_t f_0 + \nu_a \, f_0 &= G_0(f_1, \omega_b^2) \\ \partial_t f_1 + ikv \, f_1 + \nu_a \, f_1 &= G_1(f_0, f_2, \omega_b^2) \\ \partial_t f_2 + 2ikv \, f_2 + \nu_a \, f_2 &= G_2(f_1, \cancel{f_3}, \omega_b^2) \end{aligned}$$

Solve iteratively and substitute f_1 into power-balance

$$\begin{aligned} \frac{d\omega_b^2}{dt} = \underbrace{(\gamma_{L0} - \gamma_d)\,\omega_b^2}_{\text{Linear growth}} - \frac{\gamma_{L0}}{2} \int_{t/2}^{t} \mathrm{d}t_1 \int_{t-t_1}^{t_1} \mathrm{d}t_2 (t-t_1)^2 \\ e^{-\nu_a(2t-t_1-t_2)} \underbrace{\omega_b^2(t_1)\,\omega_b^2(t_2)\,\omega_b^2(t+t_2-t_1)}_{\text{Cubic nonlinearity}} \end{aligned}$$

⇒ Reduced integral equation yields the time-evolution near threshold

Numerical solutions

Berk, Breizman, Pekker '96

Normalising time with $\gamma \approx \gamma_L - \gamma_d$

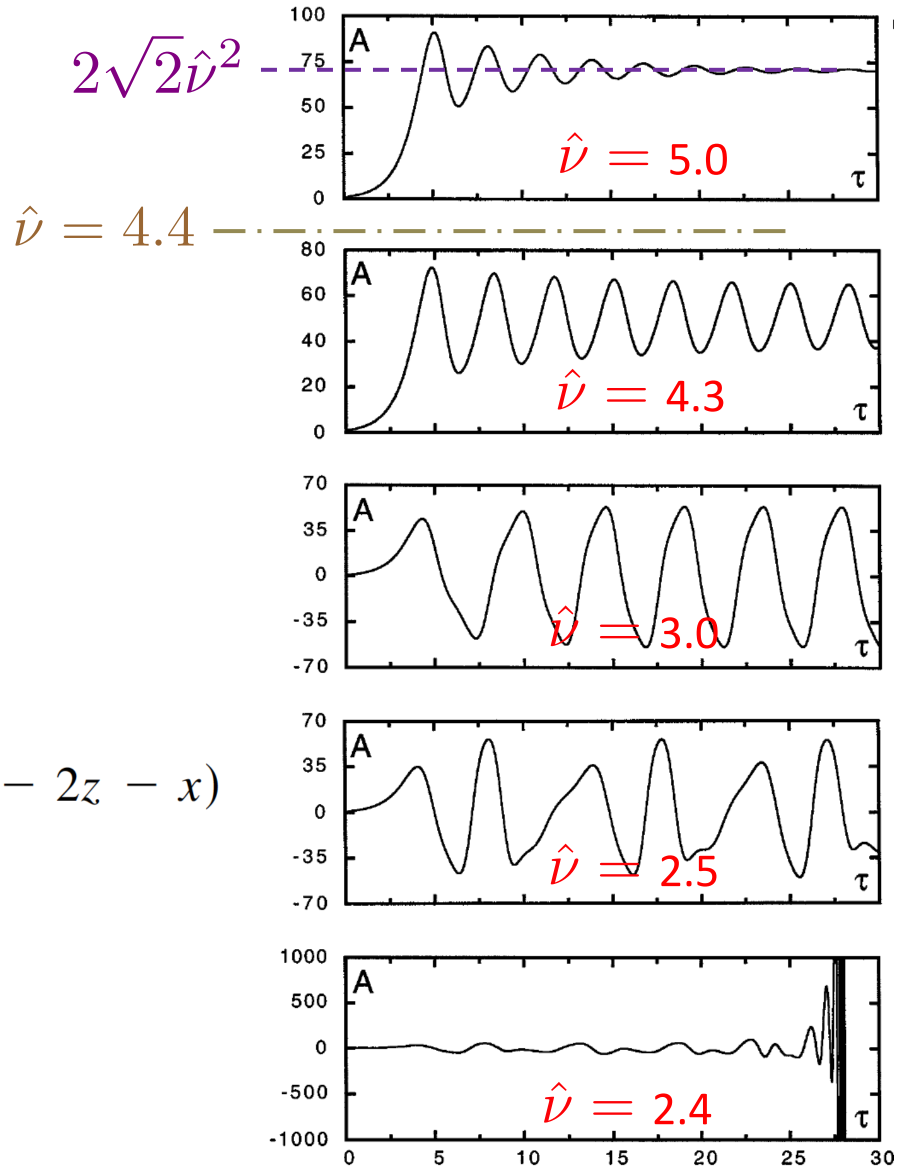
Normalising field amplitude as

$$A(\tau) = \sqrt{\frac{\gamma_L}{\gamma}}\,\frac{\omega_b^2(t)}{\gamma^2}$$

$$\frac{dA}{d\tau} = A(\tau) - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \times \int_0^{\tau-2z} dx\, \exp[-\hat{\nu}(2z + x)] \times A(\tau - z - x)A(\tau - 2z - x)$$

$\Rightarrow$ Only one parameter $\hat{\nu} = \dfrac{\nu_a}{\gamma}$

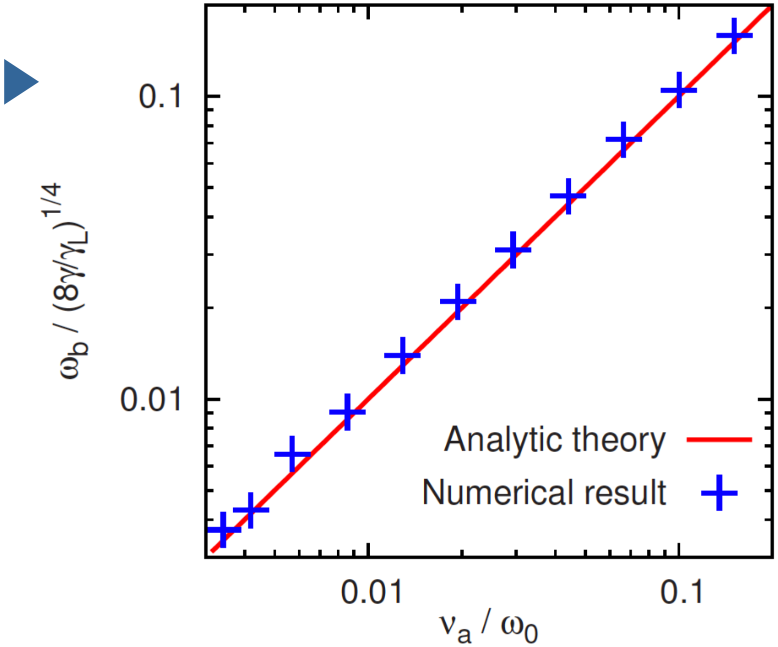
How to make sense of A<0 ?
explosive solution ?



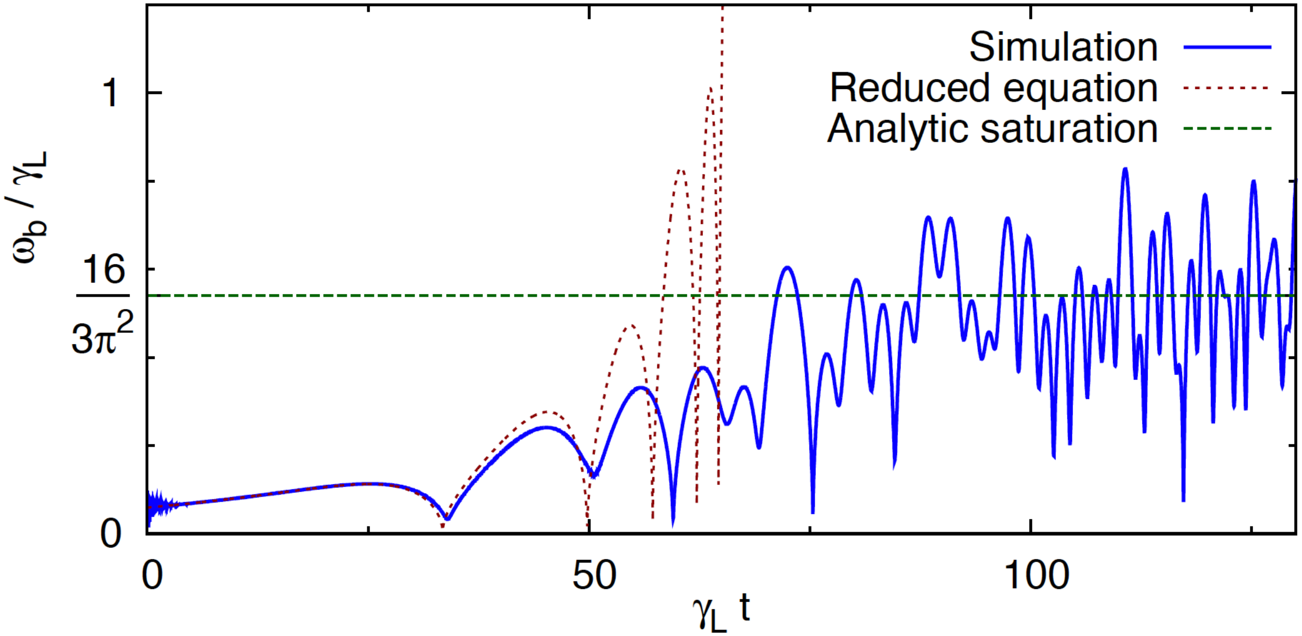
Comparisons with BB model simulation

Agreement for the saturation level ▶

(in the ideal case, but in general sensitive to f_0)



Agreement for the steady/periodic threshold (not illustrated here)



◀ Instead of explosive behavior, fast change in frequency

Bill quizz

Let’s consider a simple situation in a tokamak where energetic particles (EPs) drive a single Toroidal Alfvén Eigenmode (TAE).

Then, which one is true?

The Berk-Breizman model can qualitatively reproduce the physics of

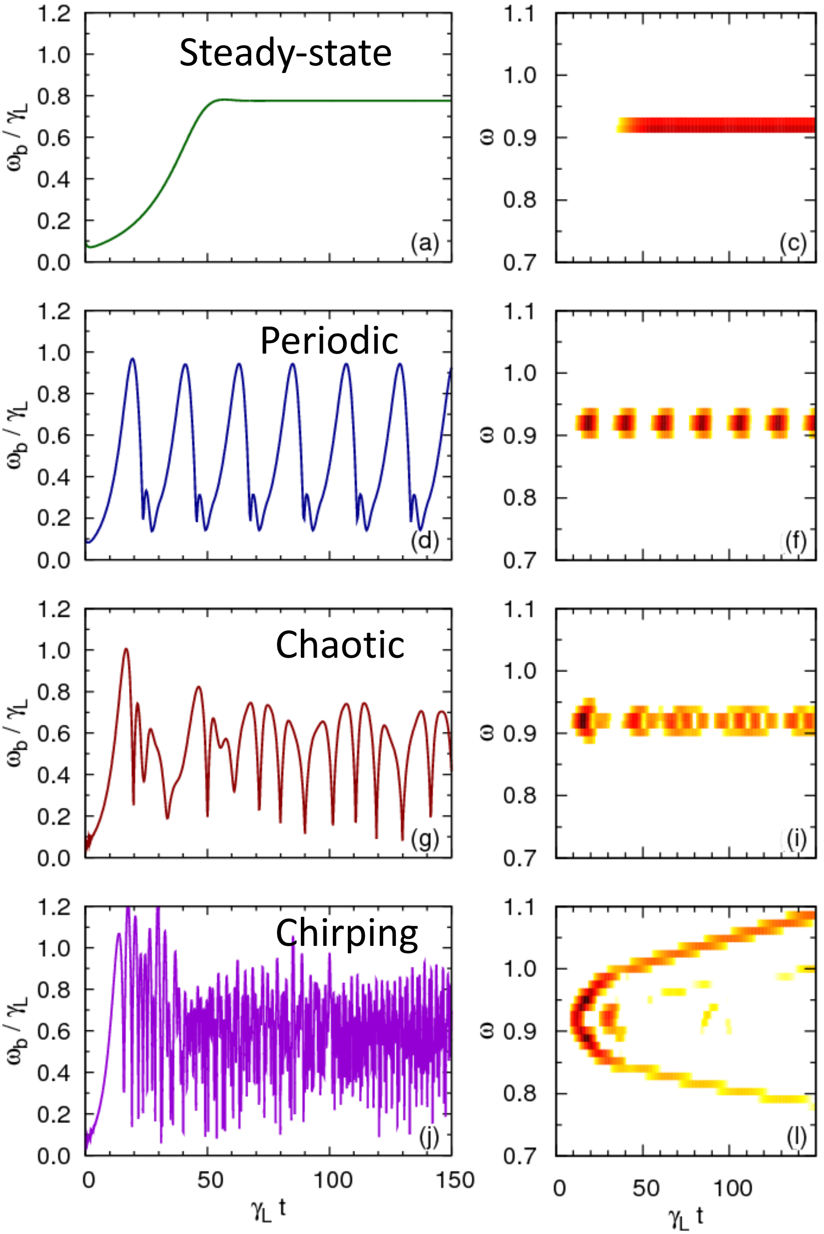
1. The slow time-evolution of the radial structure of the TAE
2. Small radial deviations of EPs due to their nonlinear interactions with the TAE
3. Nonlinear interactions between the TAE and thermal ions

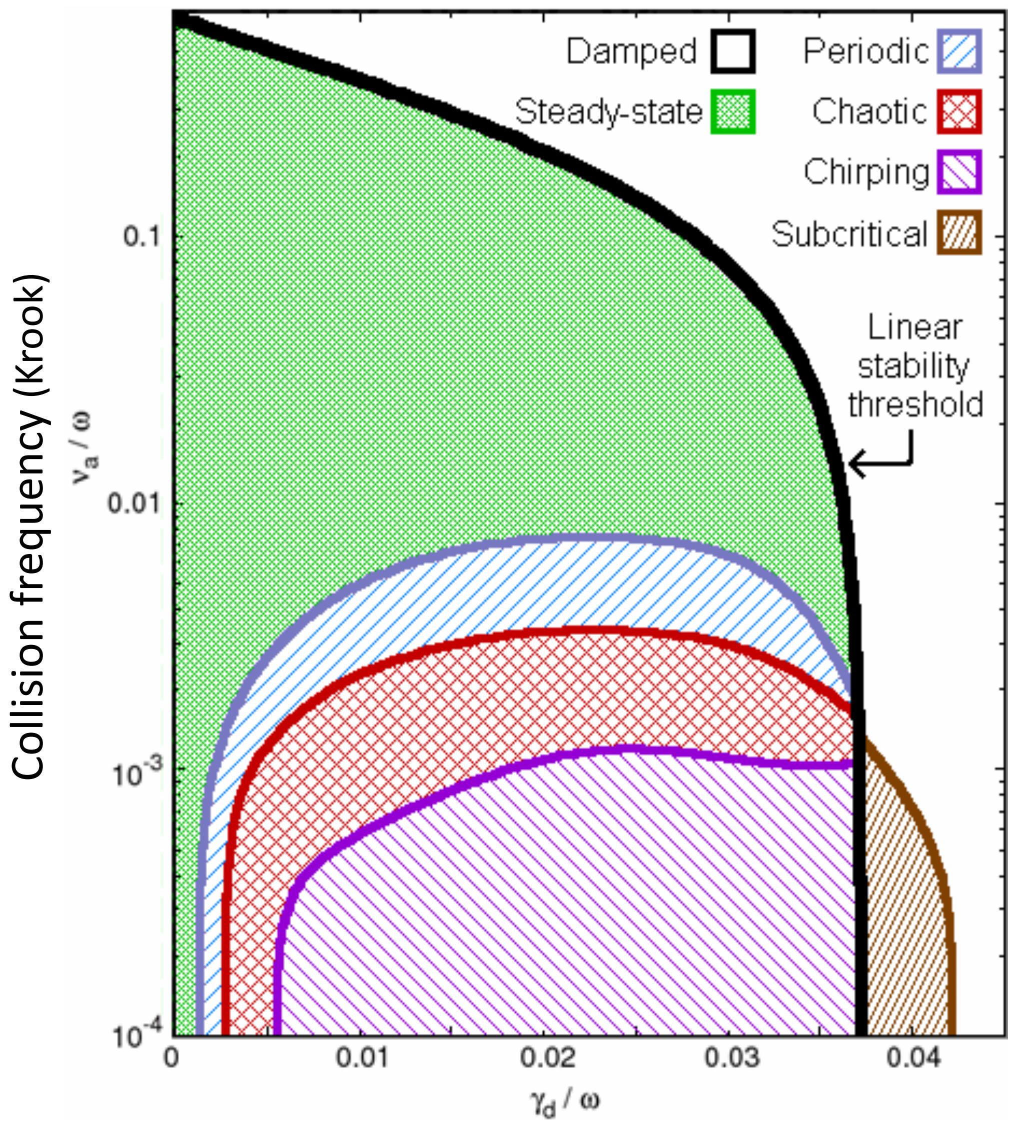
Outline

I. Bump-on-tail instability
II. The Berk-Breizman model
➡ III. Chirping
IV. Subcritical instabilities

Perspectives

Phenomenology



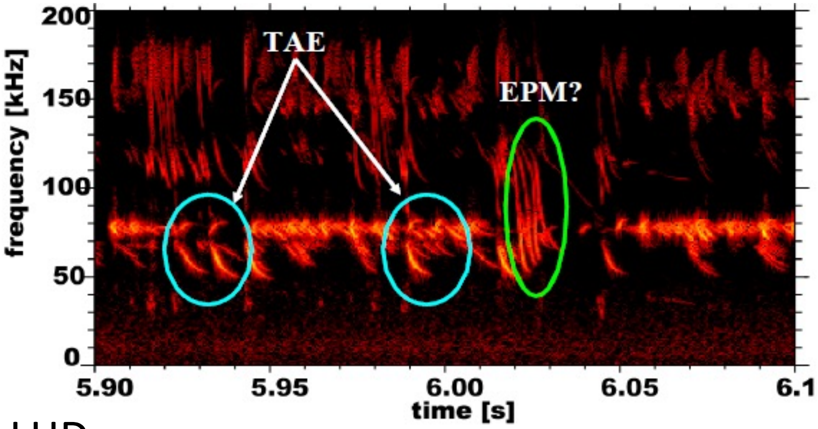


External damping rate

Chirping (fast frequency sweeping)

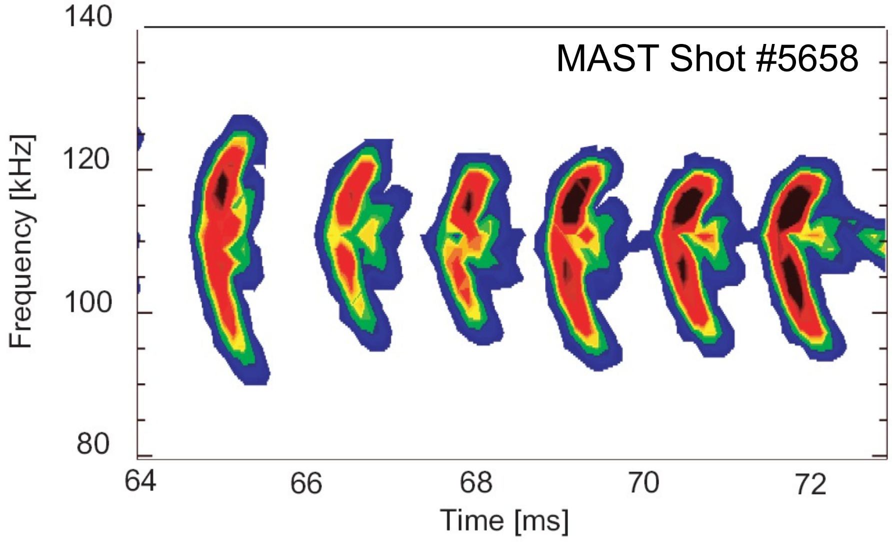
NL Chirping

- Splitting into two spectral modes, up and down in general,
- Chirping timescales << timescales of evolution of equilibrium

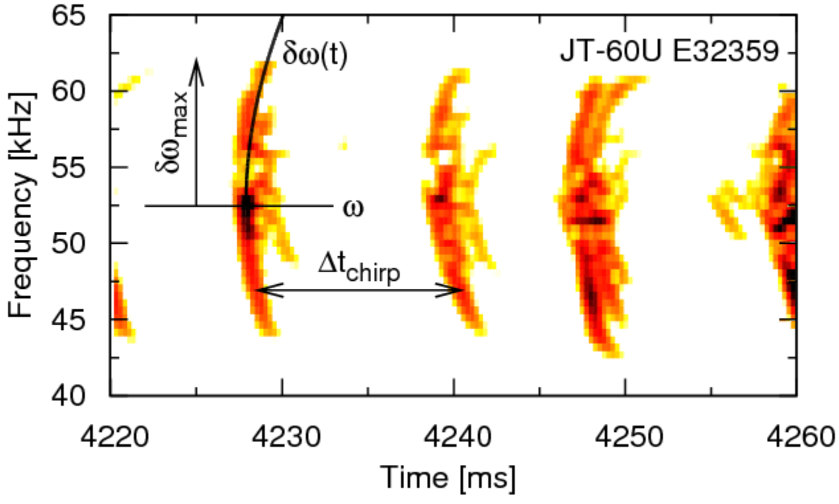


LHD

Toi, et al, PPCF(11)



Gryaznevich, Sharapov, PPCF(04)

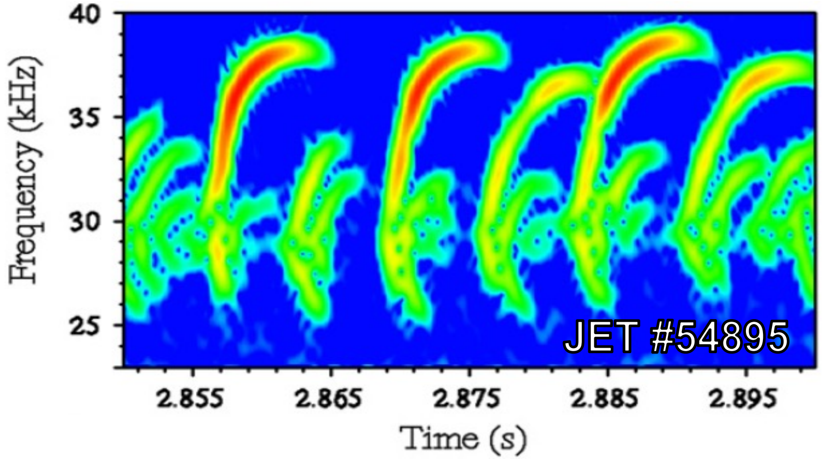


Kusama, et al., NF(99)

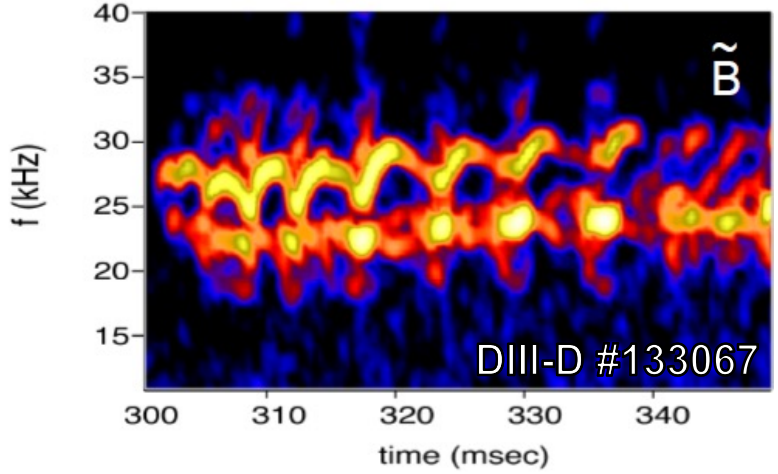
Observations of chirping TAEs

Chirping (2)

Chirping GAMs

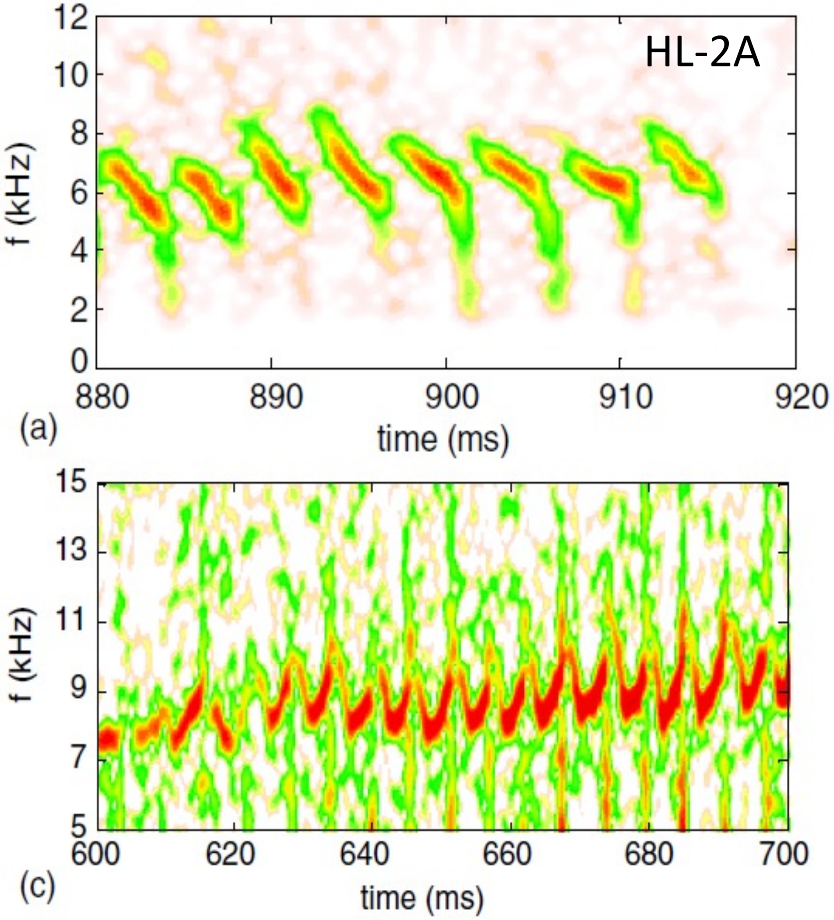


Berk, et al., NF(06)



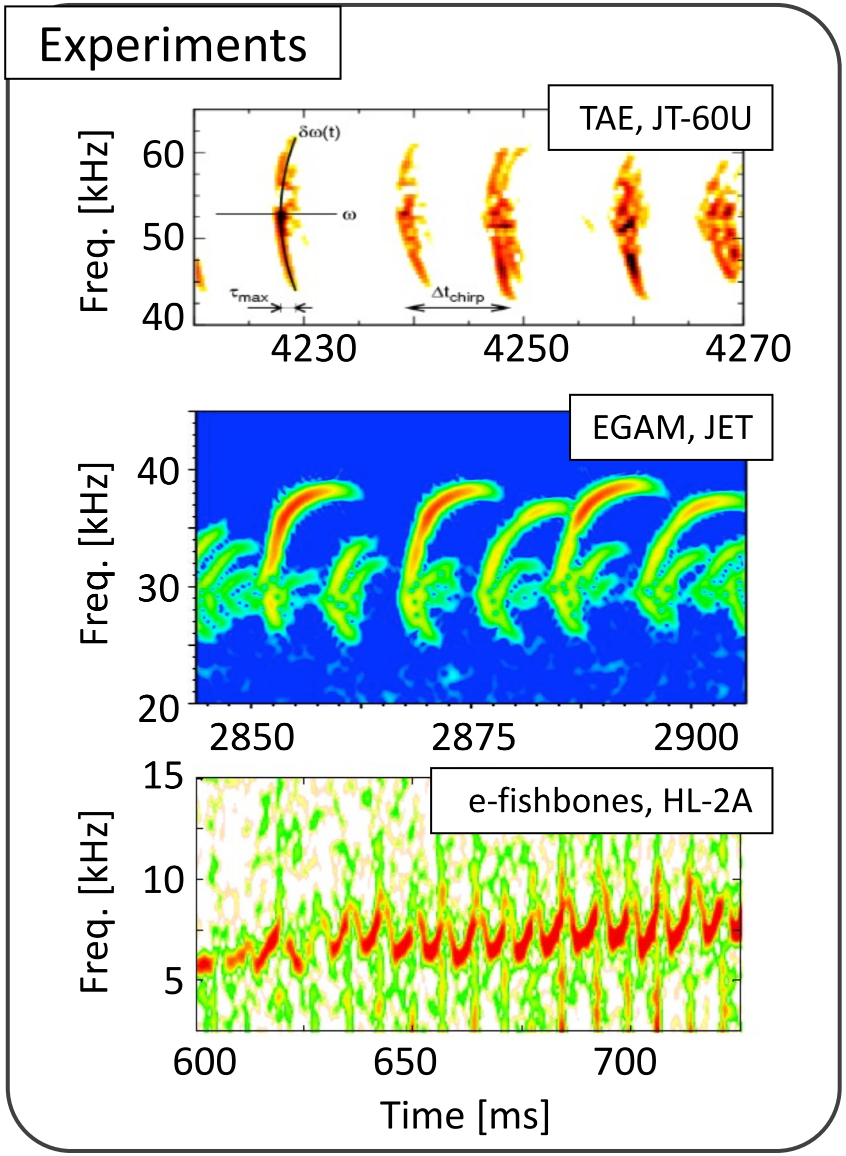
Nazikian, et al., PRL(08)

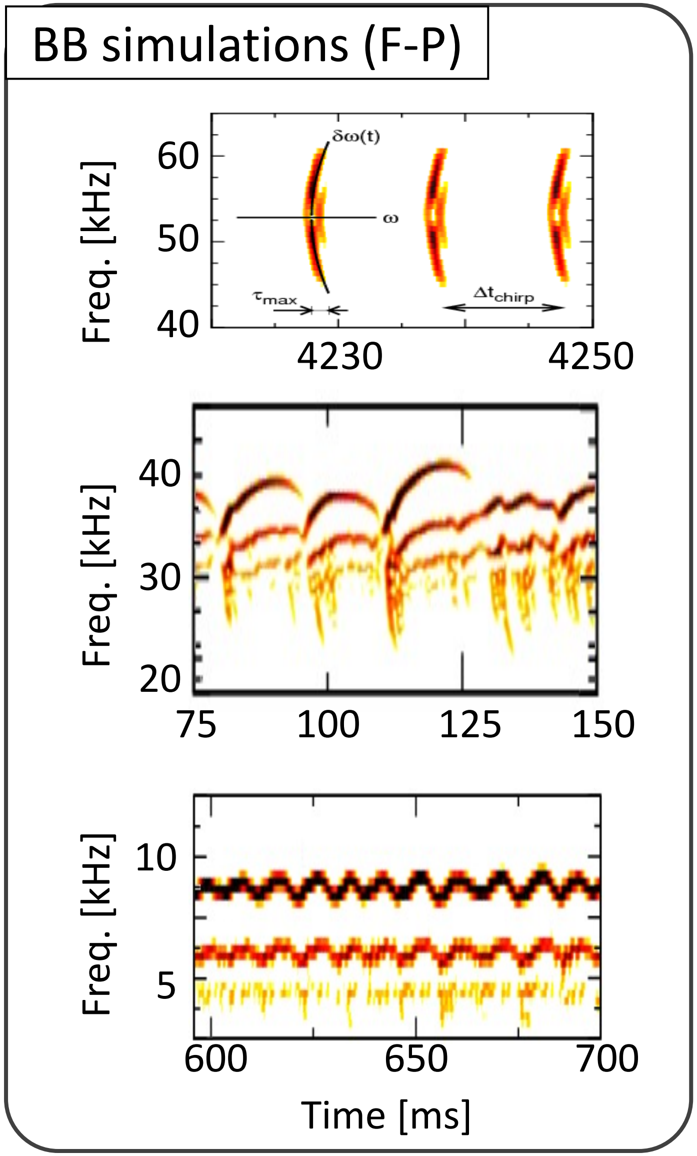
Chirping e-fishbones



Chen, et al., NF(10)

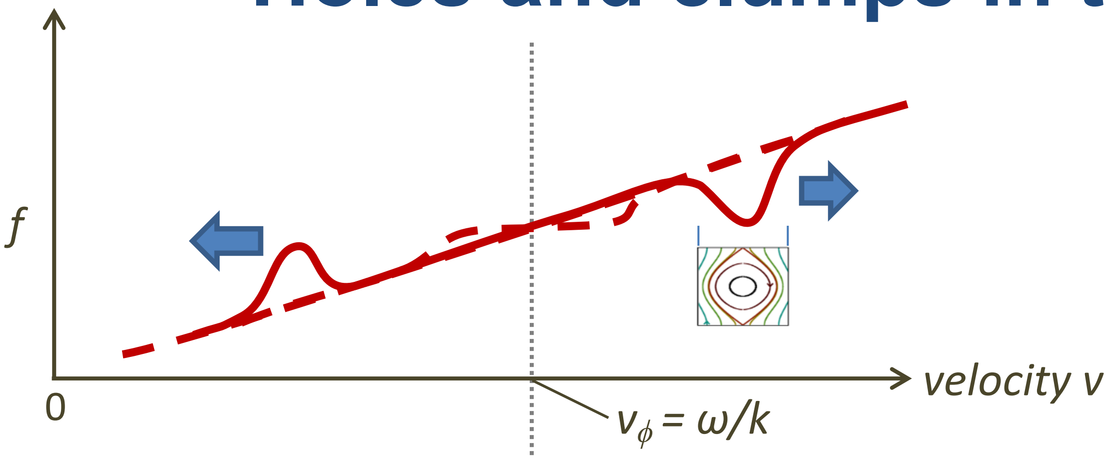
Same behavior in BB model





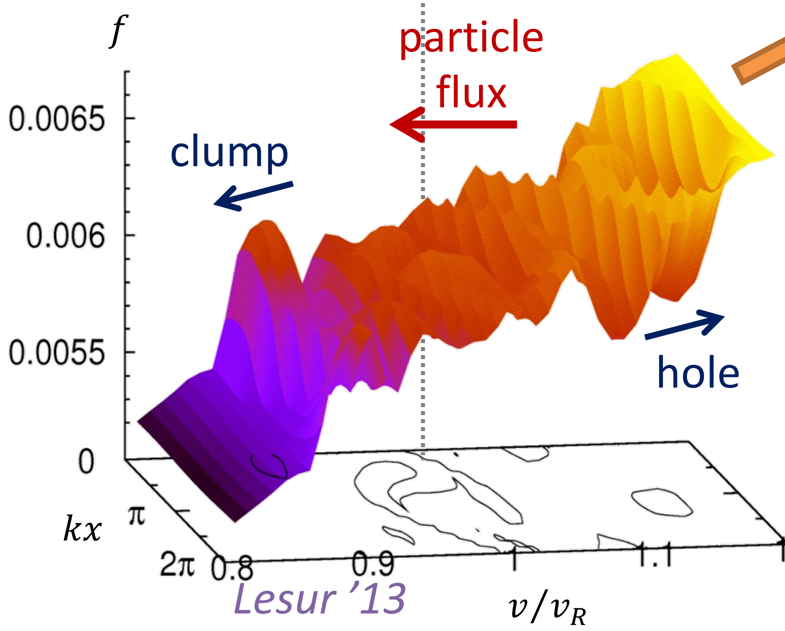
⇒ BB model to understand chirping of EP-driven modes

Holes and clumps in the BB model



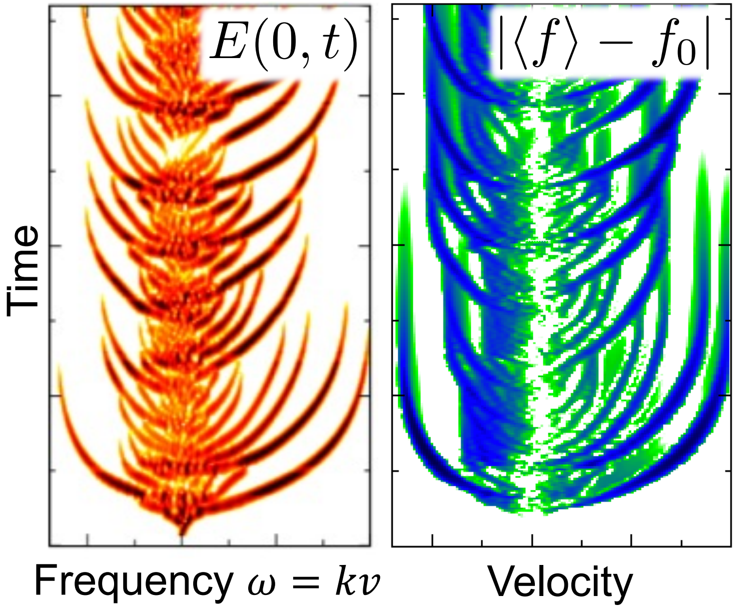
Berk, Breizman '97
Vann '05
Lesur '09

$$E(x,t) = \underset{\text{free}}{\underset{\uparrow}{E_0(t)}}\, cos(kx - \underset{\text{fixed}}{\underset{\uparrow}{\omega}} t)$$



Lesur '13

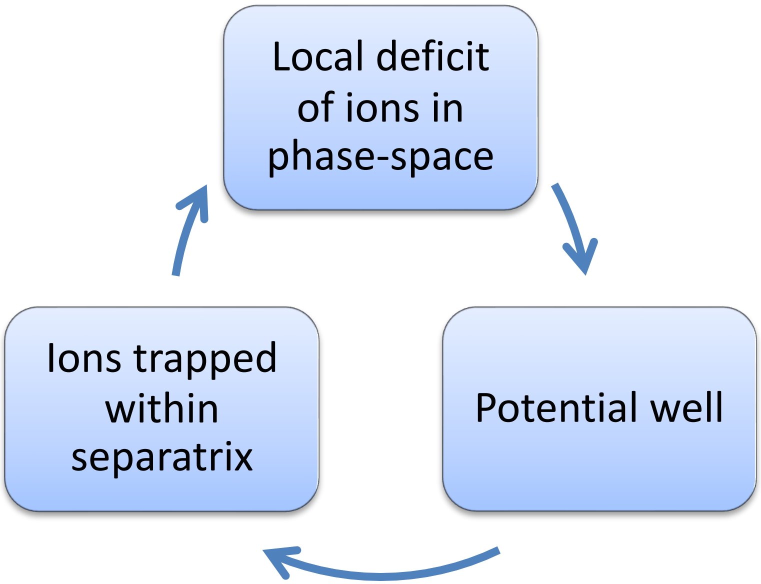
Nonlinear frequency sweeping (chirping)



⇒ Chirping (fast frequency sweeping) = fast vortex dynamics

A phase-space hole is self-coherent

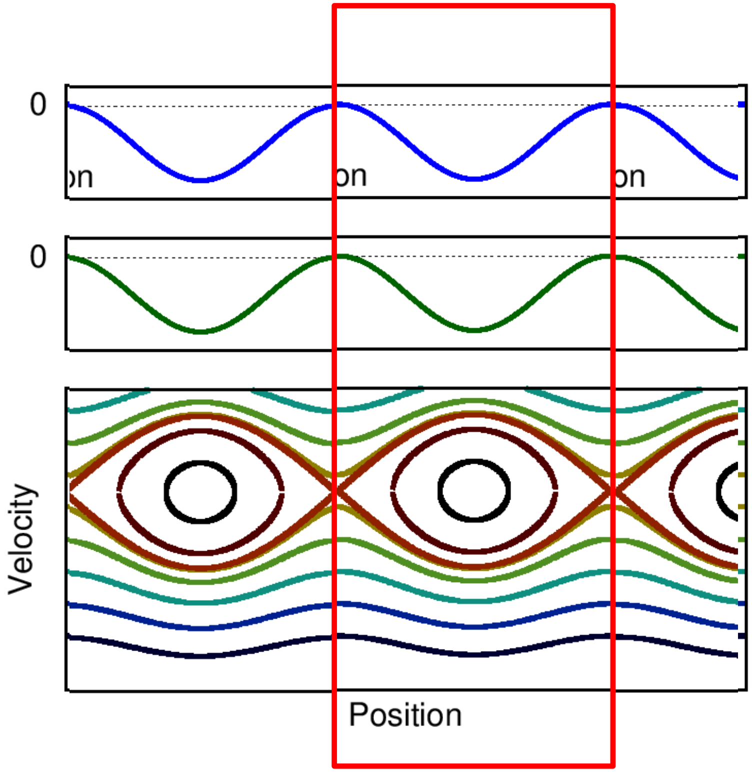
Self-sustaining structure



Local deficit of ions in phase-space

Potential well

Ions trapped within separatrix



Phase-space vortex formation is a fully nonlinear, kinetic process

⇒ BGK mode or soliton

Bernstein & Green & Kruskal ’57
Dupree ’83
Schamel ’86
Berk & Breizman ‘99

Collisions (and numerical inaccuracies) tend to fill the holes

⇒ a PS vortex is not tied to a wave and can evolve independently

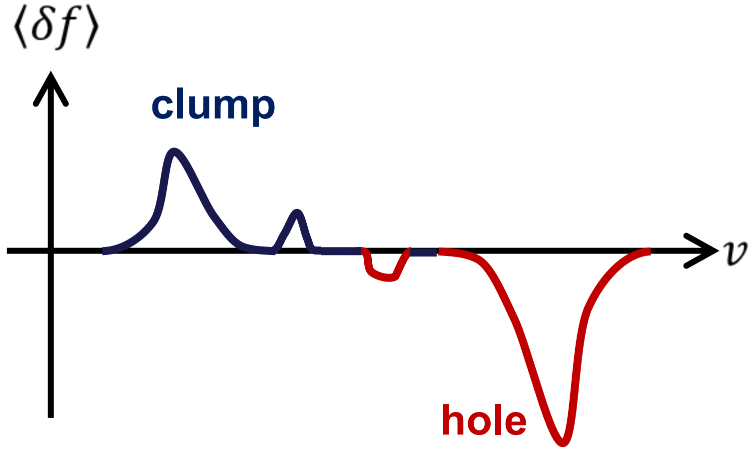
Phasestrophy and momentum exchange

Phasestrophy is the phase-space density auto-correllation, $\Psi \equiv \int \langle \delta f^2 \rangle \, \mathrm{d}v$

⇒ Measure of phase-space vortices.

Diamond, Itoh, Itoh, Modern Plasma Physics

Kosuga & Diamond ’11



Increasing phasestrophy implies:

- Structure growth
- Growth of relative entropy
- Momentum exchange

$$\frac{d\psi}{dt} = -\frac{2}{m}\frac{df_0}{dv}\frac{dp_{struct.}}{dt} \qquad p_{kin} + p_{wave} = \text{const.}$$

- Particle transport in “velocity space”

Energy/phasestrophy (W-Ψ) theorem *Lesur & Diamond, ‘13*

Wave energy

$$\frac{dW}{dt} + 2\gamma_d W = \sum_s \frac{m_s u_s}{d_v f_{0,s}} \left(\gamma_\Psi^{\text{col}} + \frac{d}{dt} \right) \Psi_s$$

Energy dissipation rate

⇒ 4 points of view: momentum, energy, entropy, phasestrophy

Nonlinear growth rate

Energy/phasestrophy (W-Ψ) theorem

$$\frac{dW}{dt} + 2\gamma_d W = \sum_s \frac{m_s u_s}{d_v f_{0,s}} \left(\gamma_\Psi^{\rm col} + \frac{d}{dt}\right)\Psi_s$$

$$\gamma_\Psi^{\rm col} = 2\nu_a + \frac{2}{\Psi_s}\frac{\nu_d^3}{k^2}\int_{-\infty}^{\infty}\left\langle\left(\frac{\partial\delta f_s}{\partial v}\right)^2\right\rangle {\rm d}v$$

Model of Gaussian hole

$$\langle\delta f\rangle = h(t)\,\exp\left[-(v-v_0(t))^2/(2\Delta v(t)^2)\right]$$

Poisson equation → wave energy

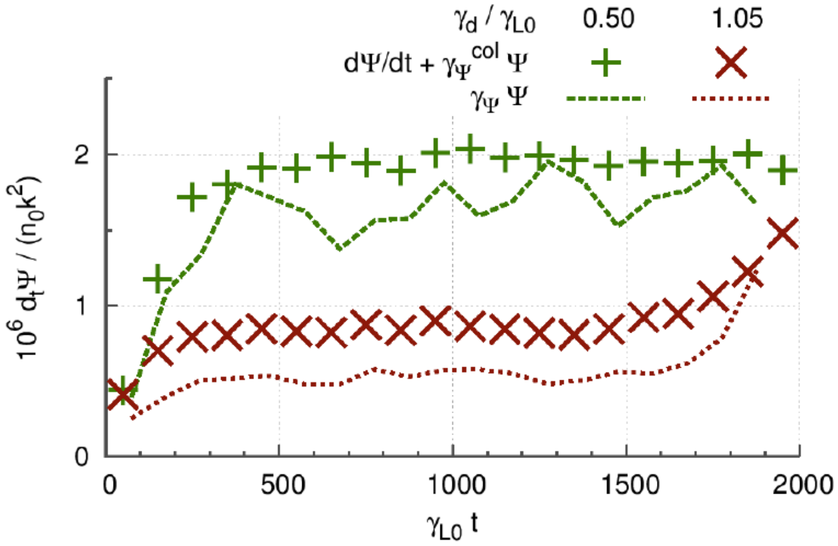
$$W = \frac{1}{2}\frac{m\omega_p^2}{k^2 n_0}\left(\int\langle\delta f\rangle\,{\rm d}v\right)^2$$

$$\Rightarrow \frac{{\rm d}\Psi}{{\rm d}t} = \left(\gamma_\Psi - \gamma_\Psi^{\rm col}\right)\Psi$$

with

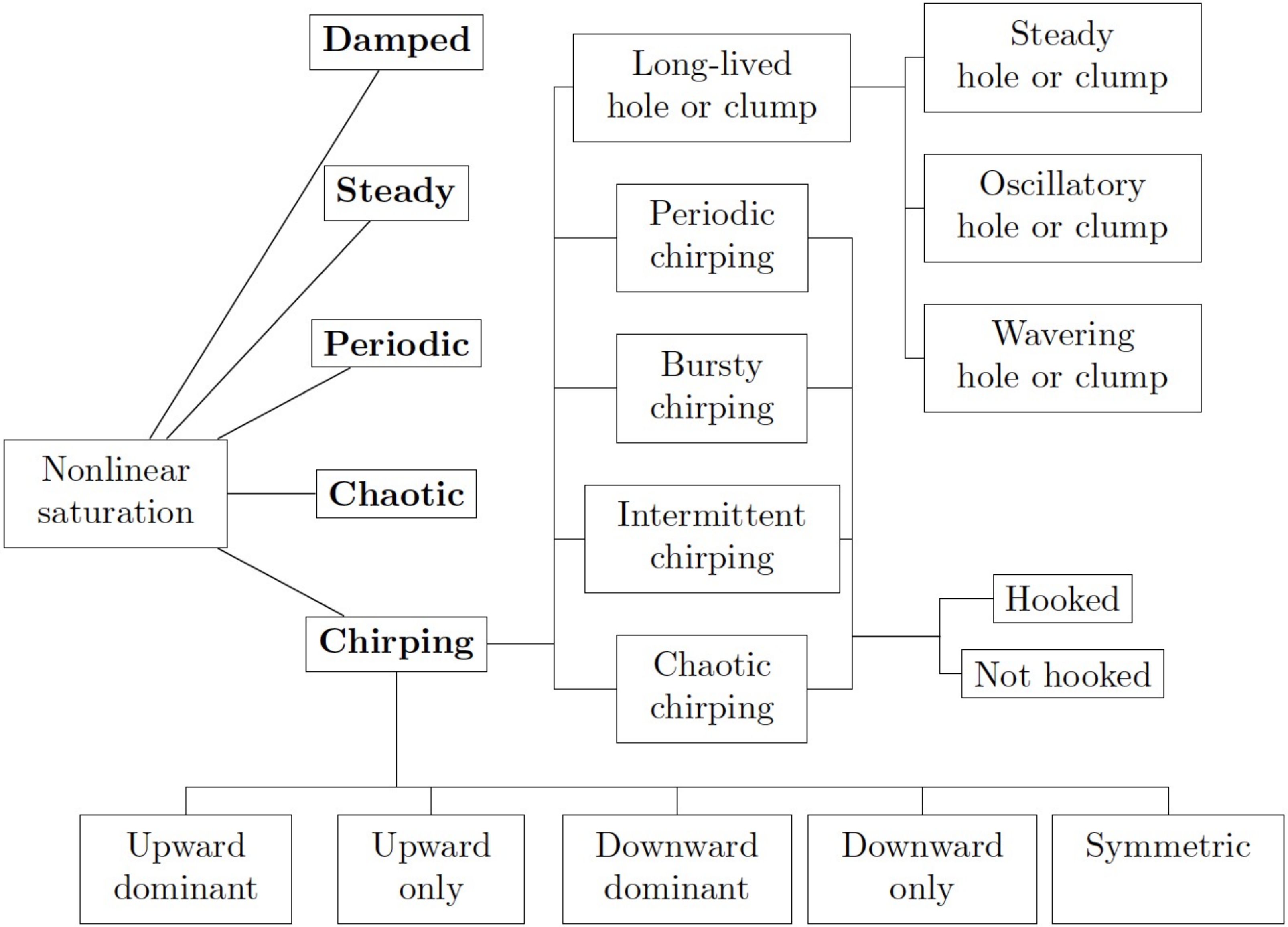
$$\boxed{\gamma_\Psi \approx \frac{16}{3\sqrt{\pi}}\frac{\Delta v}{v_R}\frac{\gamma_L}{\omega_p}\gamma_d}$$

Lesur & Diamond, '13

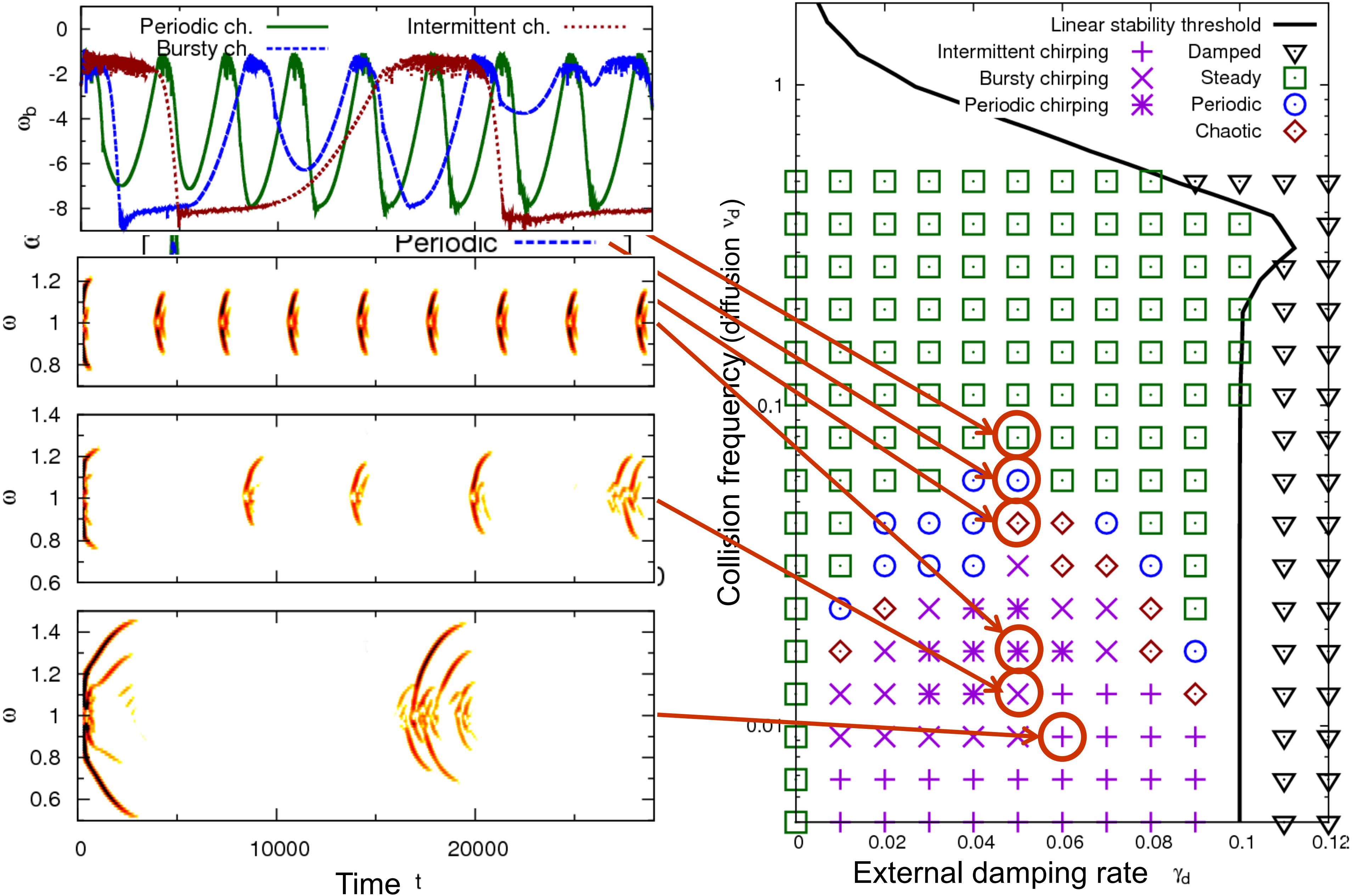


⇒ Nonlinear growth requires both free energy and energy dissipation

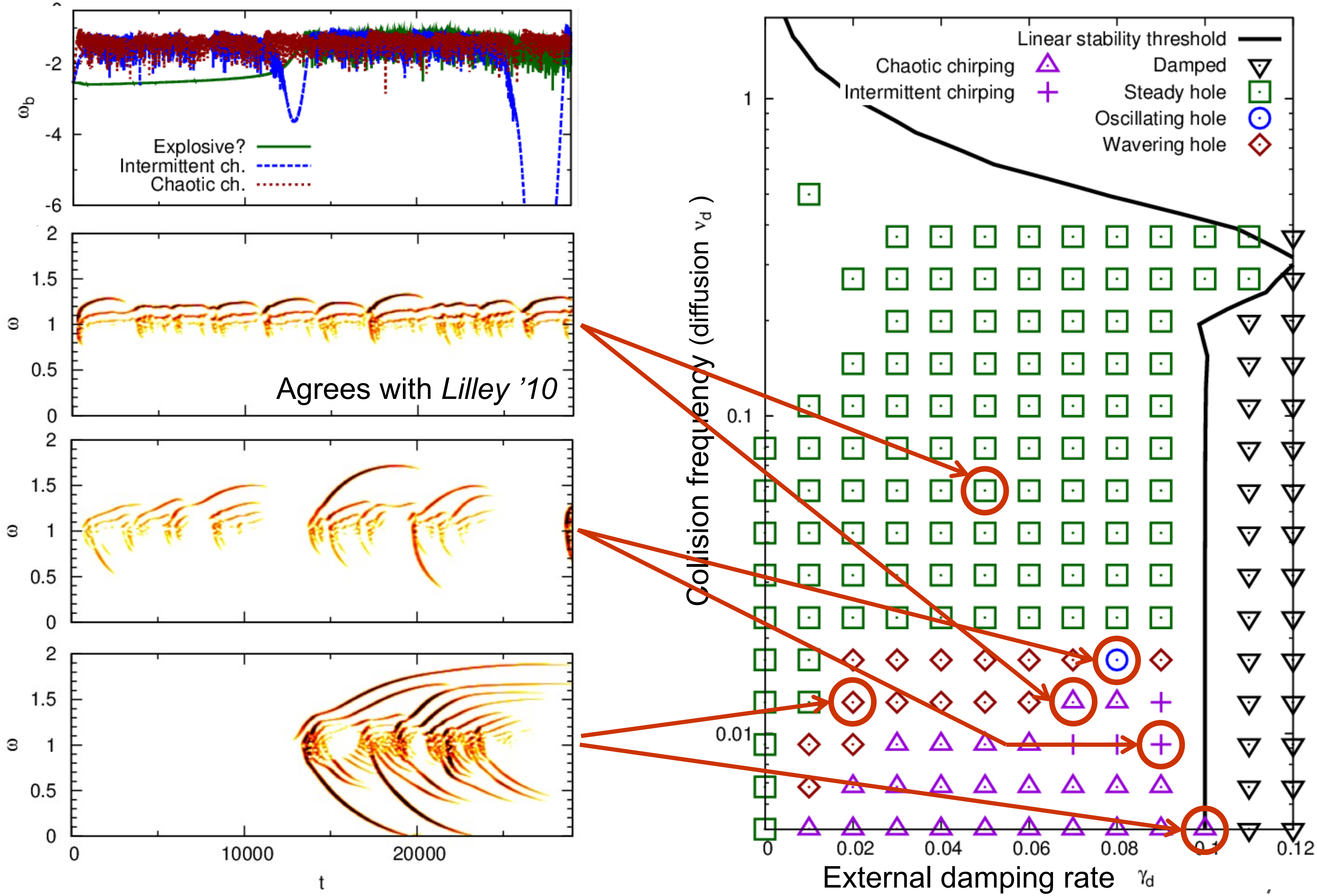
Chirping can be further categorized



Phenomenology: for small drag ($\nu_d/\nu_f = 5$)



Phenomenology: for large drag (ν_d/ν_f =1)



Agrees with *Lilley '10*

Fitting the BB model to experiments

γ_L	γ_d	ν_f	ν_d	γ
9.9%	5.0%	0.55%	2.3%	5.5%
+0.4	+0.5	+0.03	+0.05	+0.1
-0.2	-0.2	-0.05	-0.05	-0.1

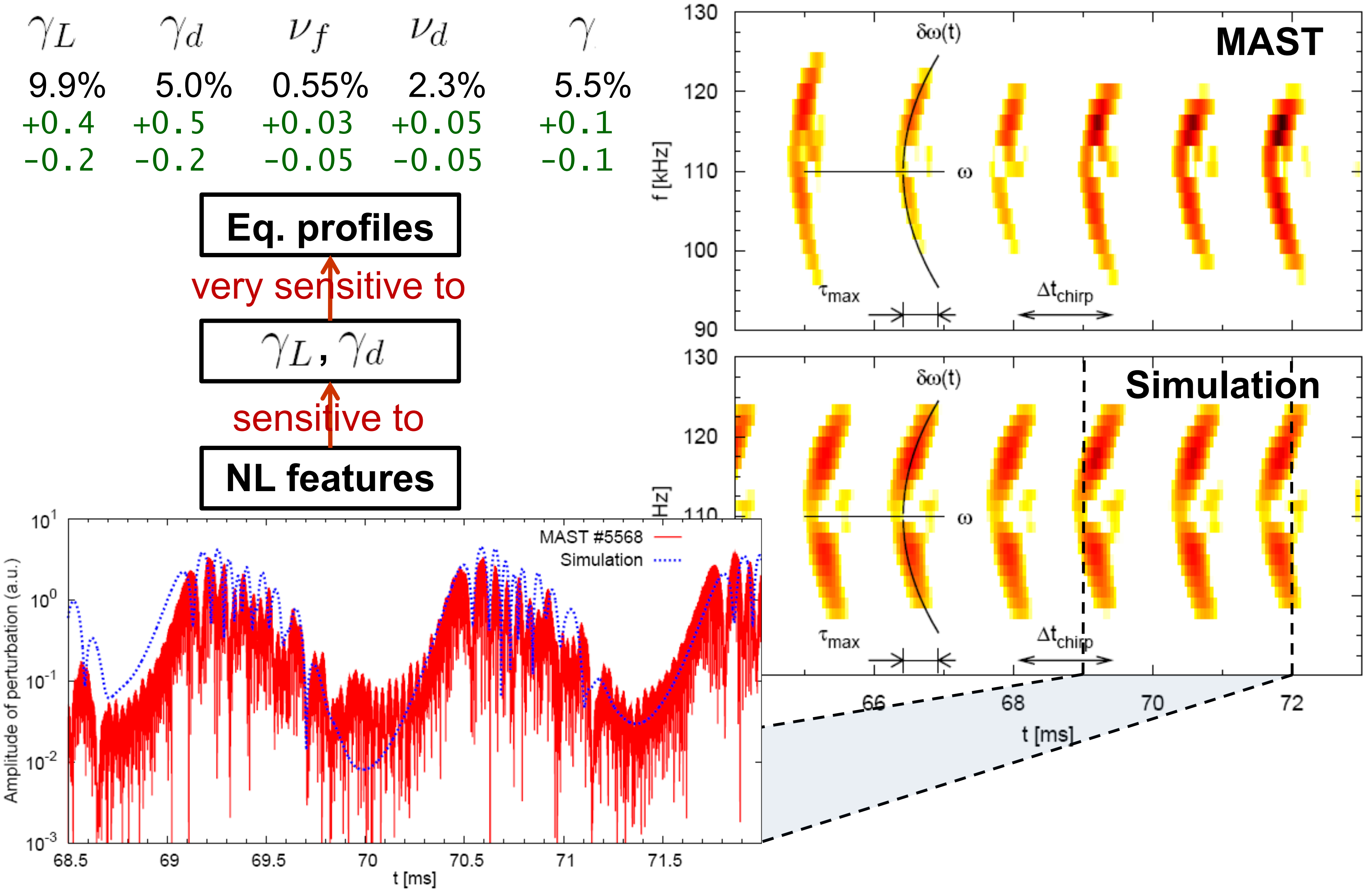
Eq. profiles

↑ very sensitive to

$\boxed{\gamma_L, \gamma_d}$

↑ sensitive to

NL features



⇒ Quantitative agreement for chirping dynamics

Chirping velocity

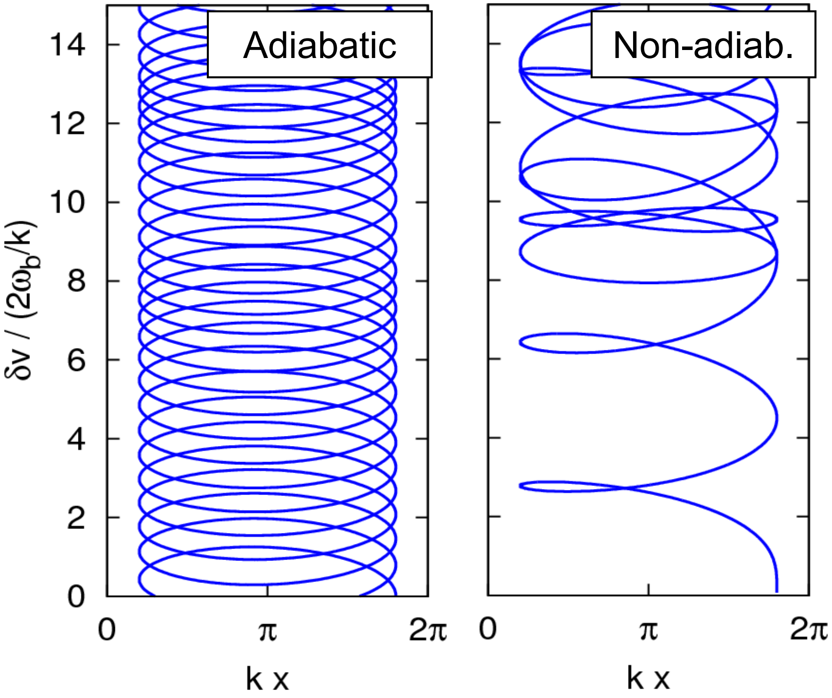
Isolated, BGK-like mode

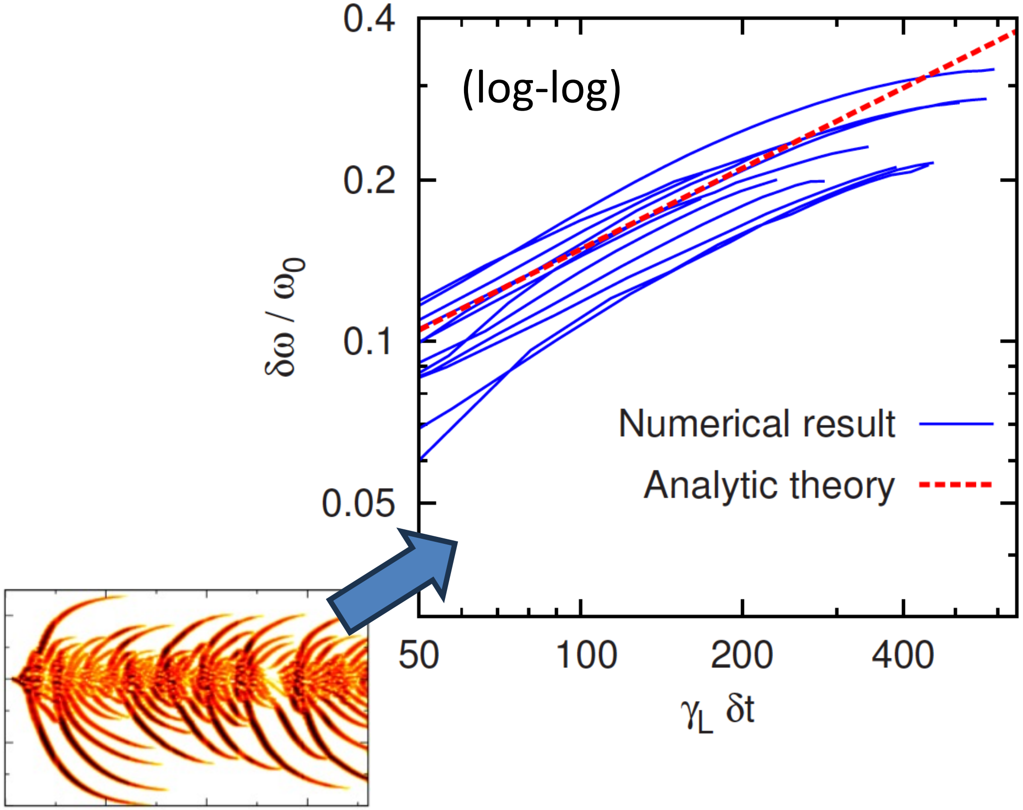
Angle-action variables

Adiabatic evolution ($\delta\dot{\omega}/\omega_b^2 \ll 1$)

$$\Rightarrow \quad \delta\omega(t) \approx 0.44\,\gamma_L\sqrt{\gamma_d\,t}$$

Berk, Breizman, et al. ’99





⇒ Nonlinear rate of change of frequency depends on linear parameters

+ saturation level

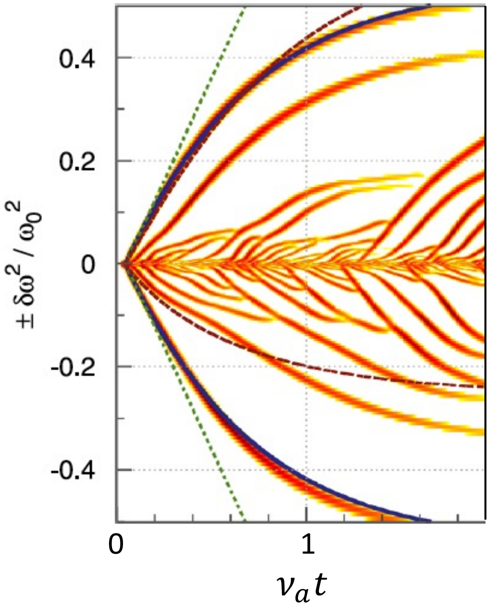
$$\frac{\omega_b}{\gamma_L} = 0.54$$

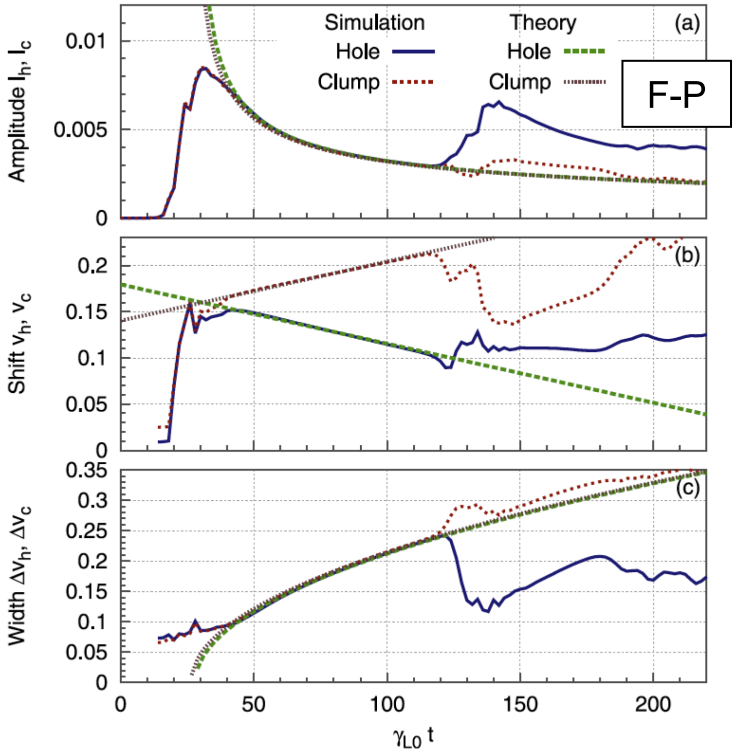
Chirping velocity (advanced)

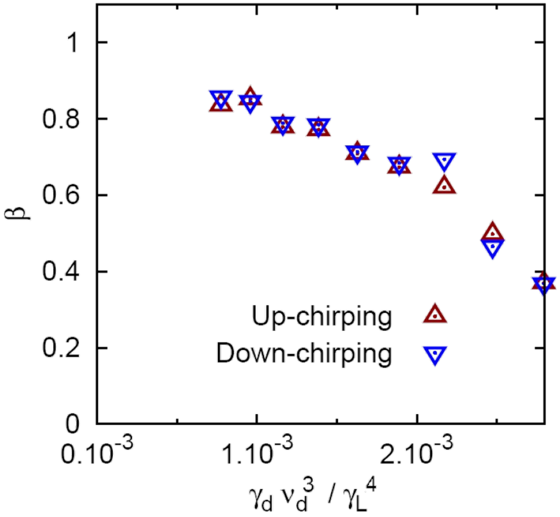
Deviations from $\delta\omega(t) \approx 0.44\,\gamma_L\sqrt{\gamma_d\,t}$

Lilley '10
Nyqvist '12
Lesur '10 and '13

Strong correction due to collisions: still analytical



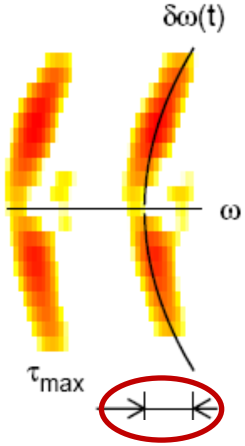




When $\dot{\delta\omega}/\omega_b^2 \sim 1,$ correction parameter $0.44 \to 0.44\,\beta$

⇒ Subtleties but frequency varying as roughly $t^{1/2}$ is a good signature

Chirping lifetime



Hole/clump width $\sim \gamma_L$

$$\Rightarrow \text{diffusion time } \tau_{max} \sim \frac{\gamma_L^2}{\nu_d^3}$$

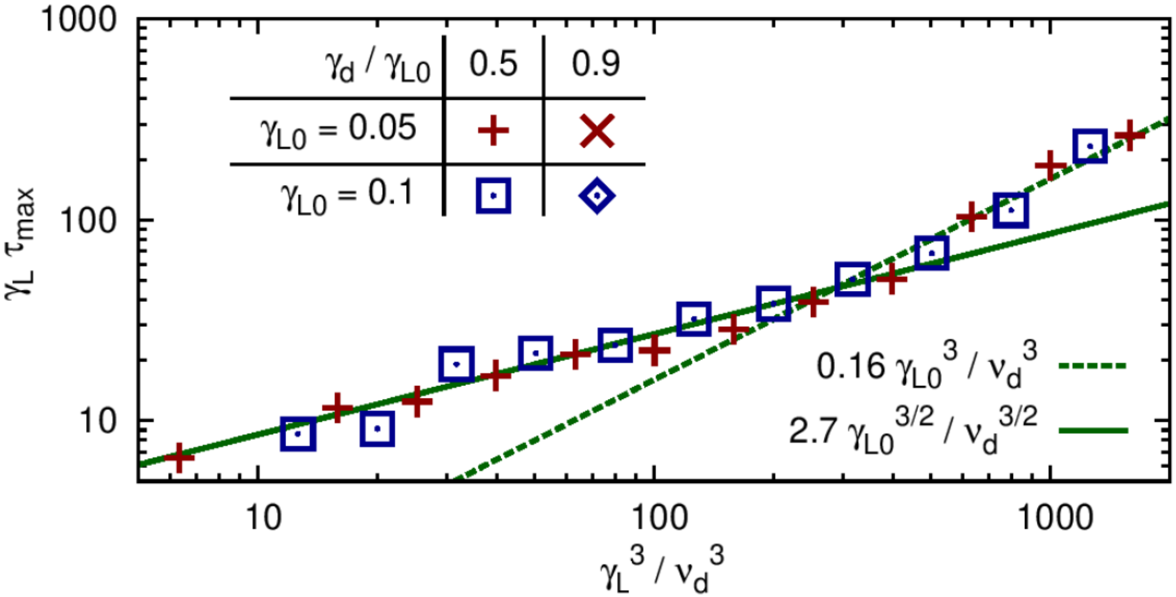
Berk, Breizman, et al., '97

However, for higher collisionality, diffusion affects hole/clump width

$\Rightarrow$ semi-empirical law

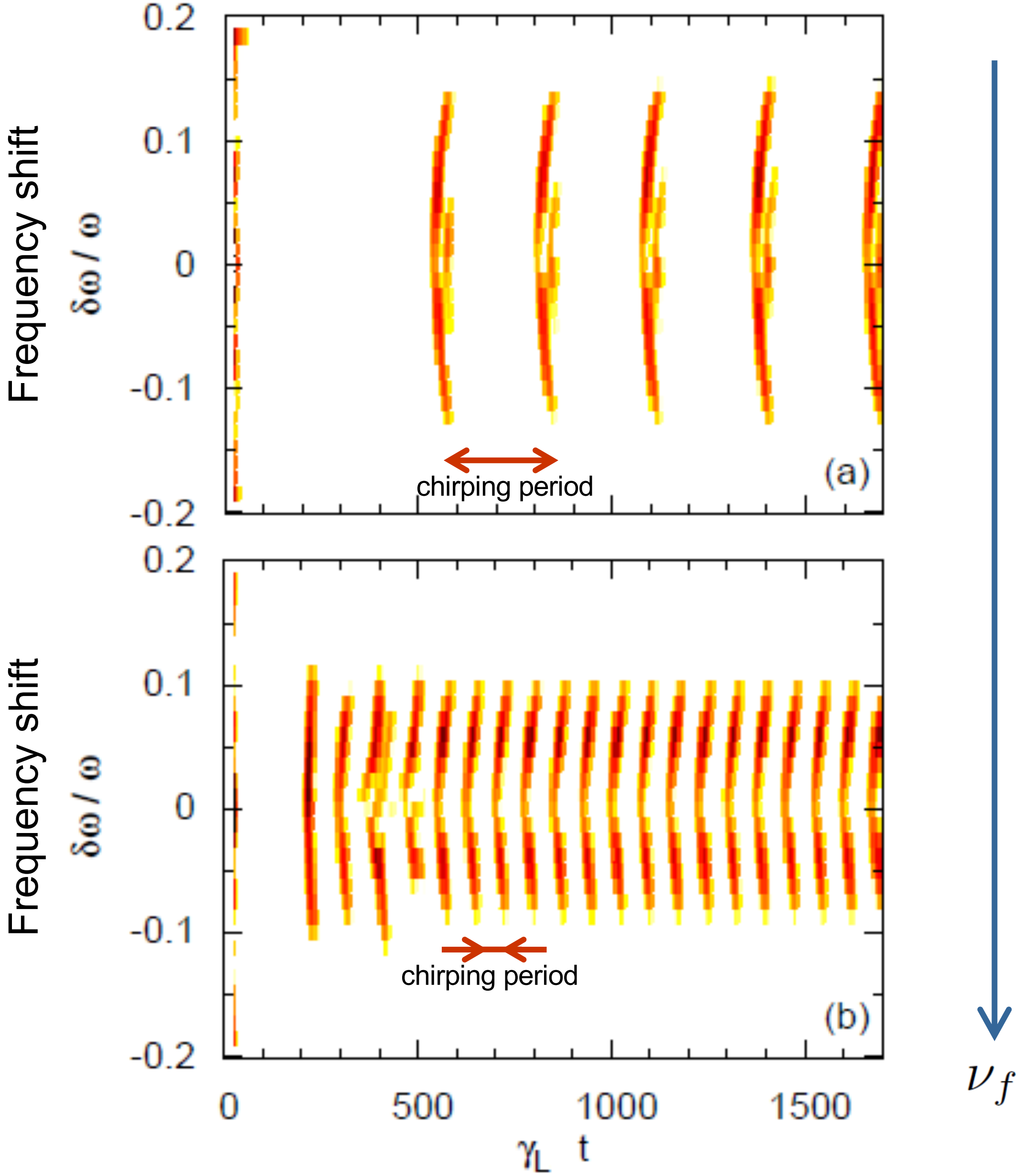
$$\tau_{max} \sim \frac{\gamma_L^{1/2}}{\nu_d^{3/2}}$$

Lesur, '20



$\Rightarrow$ Simple expression for chirping lifetime

Chirping period



In experimentally-relevant conditions, the time between 2 chirping bursts decreases with:

- Increasing ν_f
- Increasing ν_d
- Increasing γ_L

* rough picture
More details in
Lesur '13

⇒ **Strong link between velocity diffusion & frequency of bursts**

δf vs full-*f* model

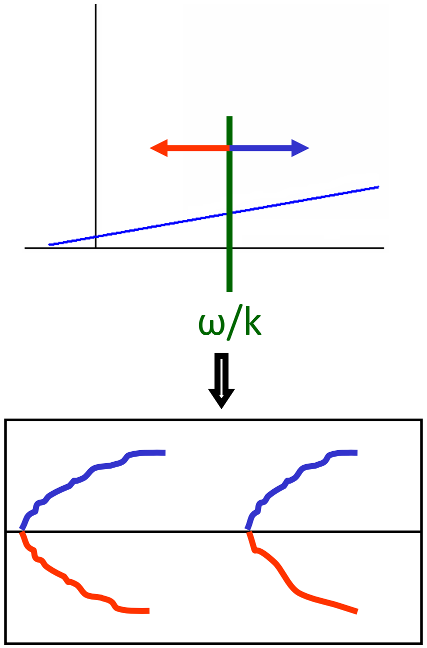
δf model

Symmetric chirping, due to:

- Constant gradient distribution

- The bulk part determines the frequency of the mode only.

- We construct a distribution such that chirping does not suffer any border effect.



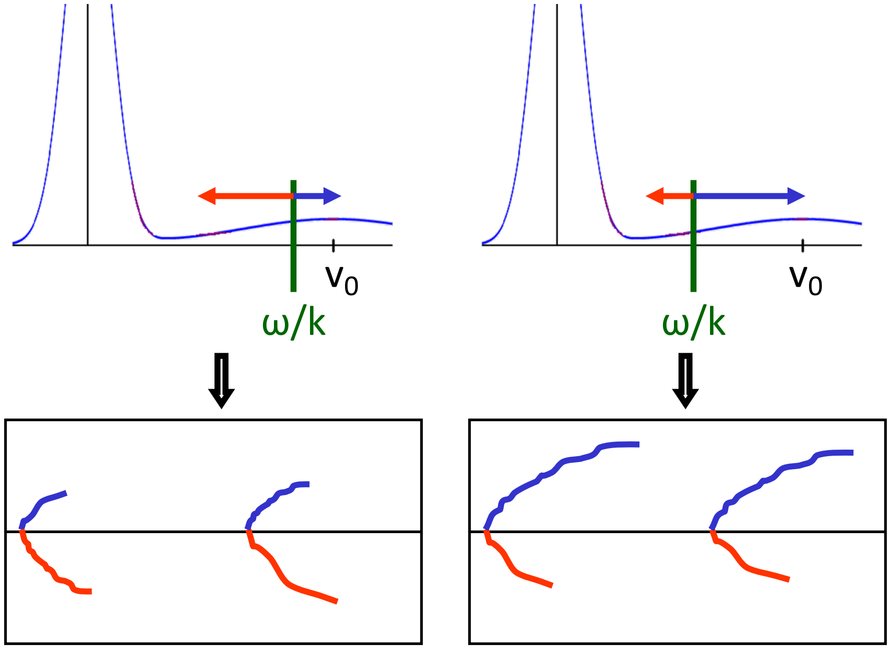
Full-*f* model

Chirping asymmetry, due to:

- Shape of the distribution

- Modification of the bulk distribution

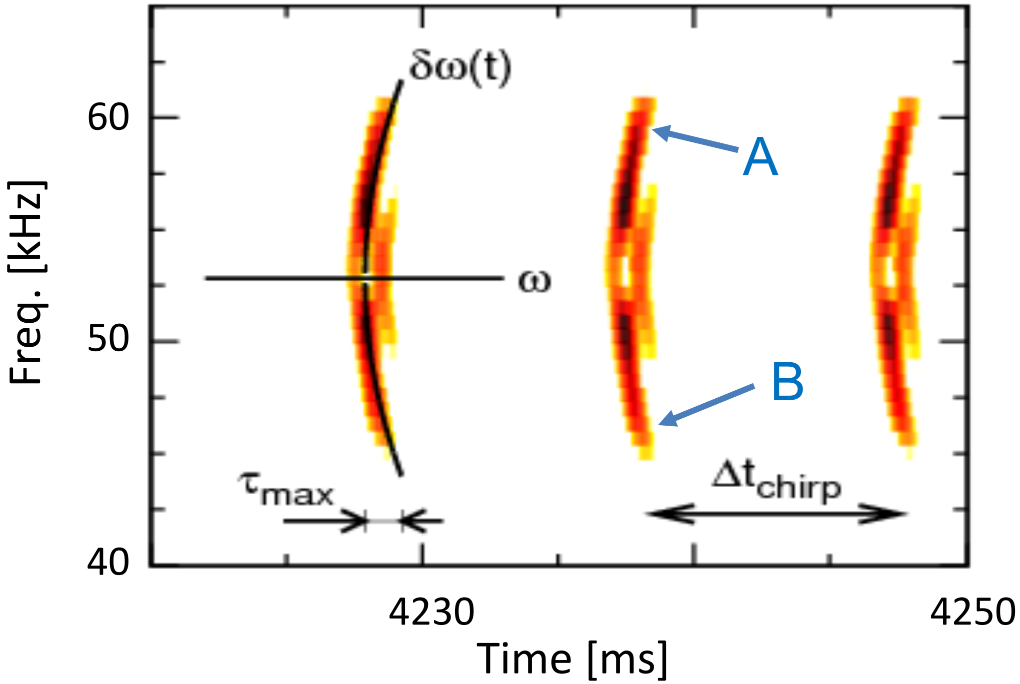
- Proximity between resonant velocity and beam velocity or bulk



Bill quizz

In a simulation of the BB model, you obtain this spectrogram of electric field fluctuations.

What do the branches A and B correspond to in the velocity distribution of fast particles?



1. Two holes
2. A is a hole, B is a clump (or bump)
3. B is a hole, A is a clump (or bump)
4. It's impossible to know without looking at the velocity distribution

Outline

I. Bump-on-tail instability

II. The Berk-Breizman model

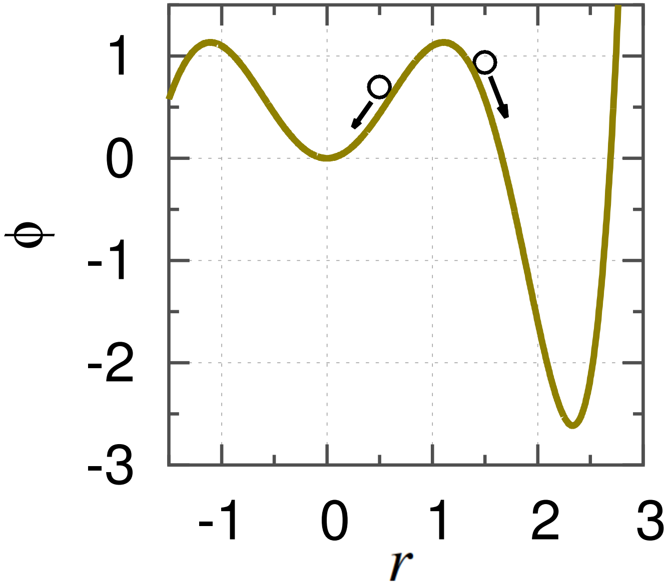
III. Chirping

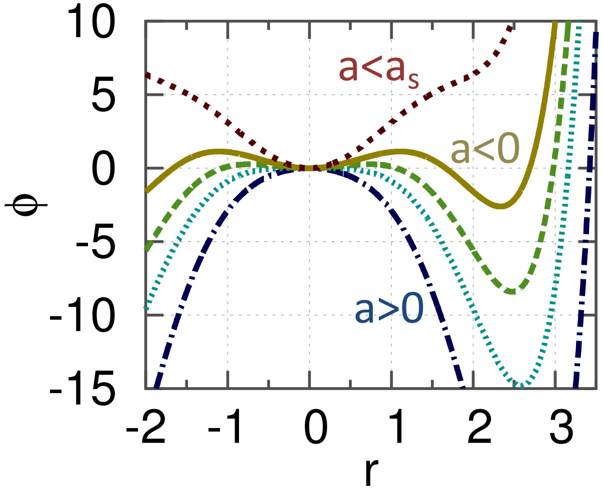
➡ IV. Subcritical instabilities

Perspectives

Basic concepts of subcritical instability

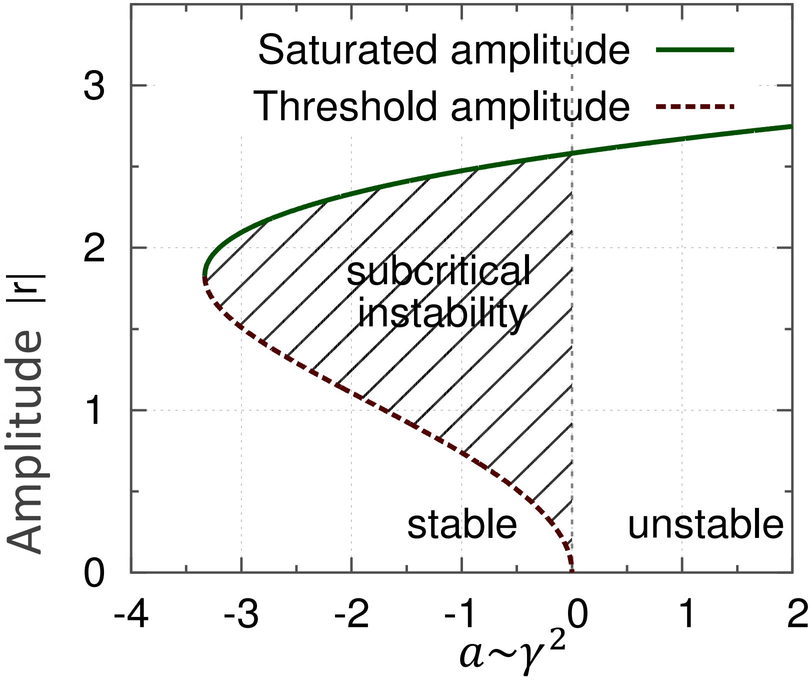
$$\phi(r) \;=\; -ar^2 - r^4 + cr^6$$





Linearized eqn of motion

$$\ddot{r} \sim ar$$



- Circumvent linear stability theory
- Threshold in amplitude
- Hysteresis
- Local vs Global

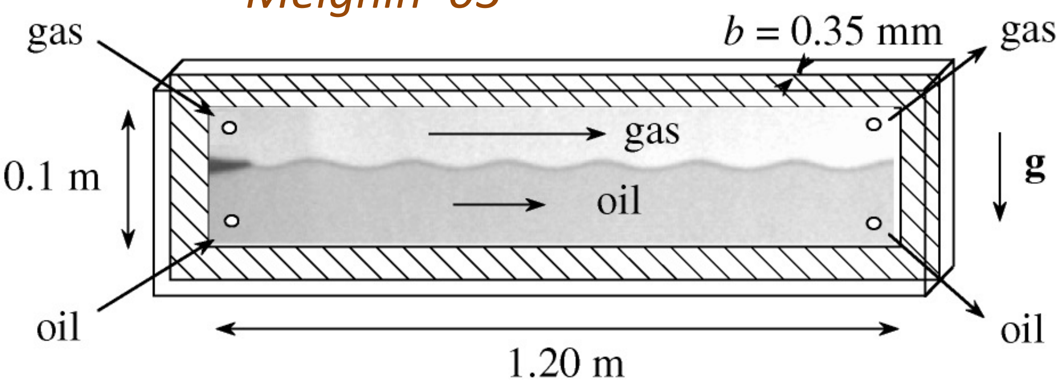
Dauchot & Manneville '97

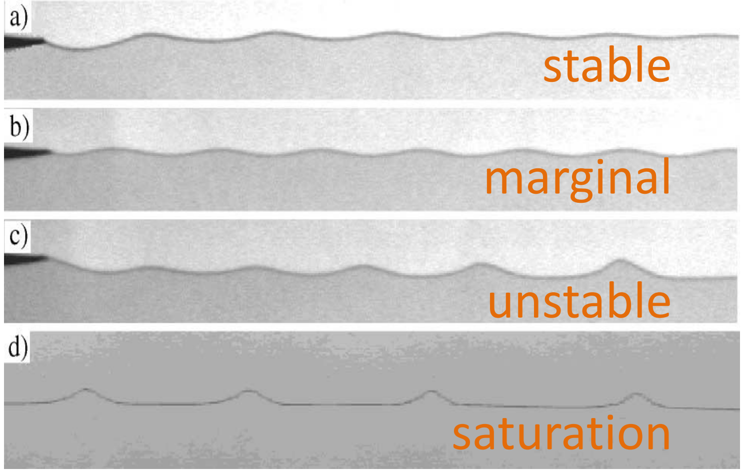
Tutorial: Lesur '18

⇒ Subcritical instabilities ubiquitous in plasmas and neutral fluids

Experimental example: Kelvin-Helmholtz

Meignin '03

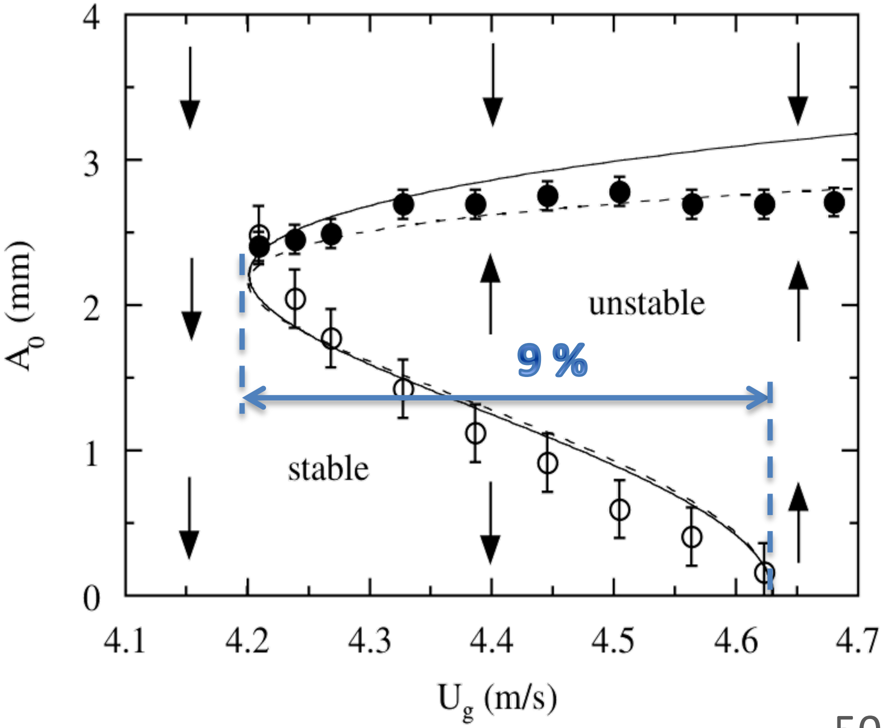




Saturated state:

Self-organized,
self-sustaining
nonlinear
structures

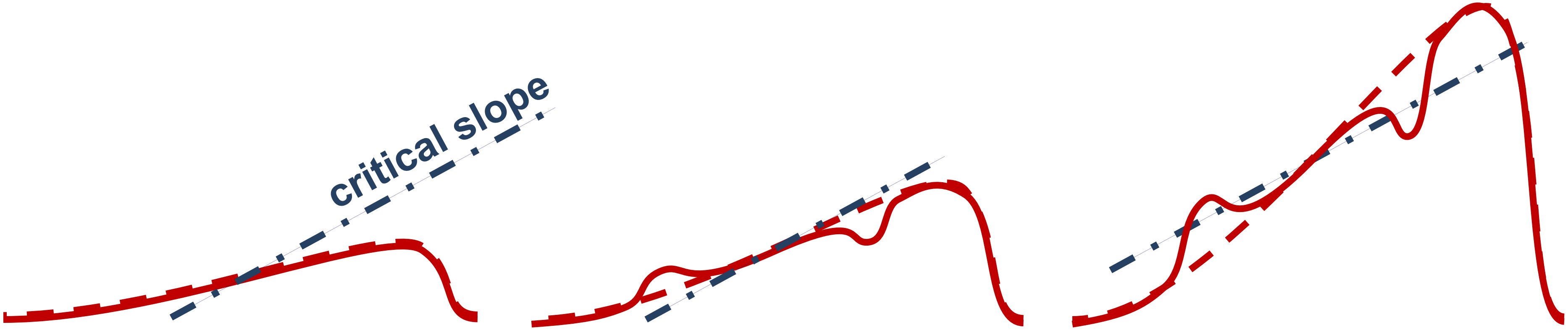
(Phase-locking)



Basic picture of stability

Linear stability

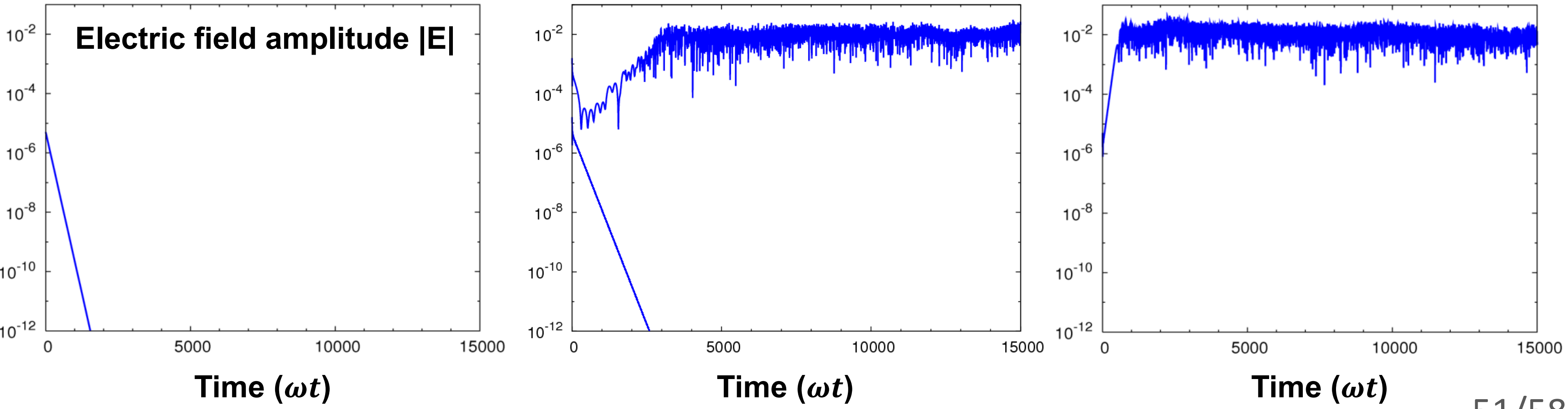
Growth rate $\gamma \approx \gamma_L - \gamma_d \;\Rightarrow\;$ Critical slope $\gamma_L = \gamma_d$



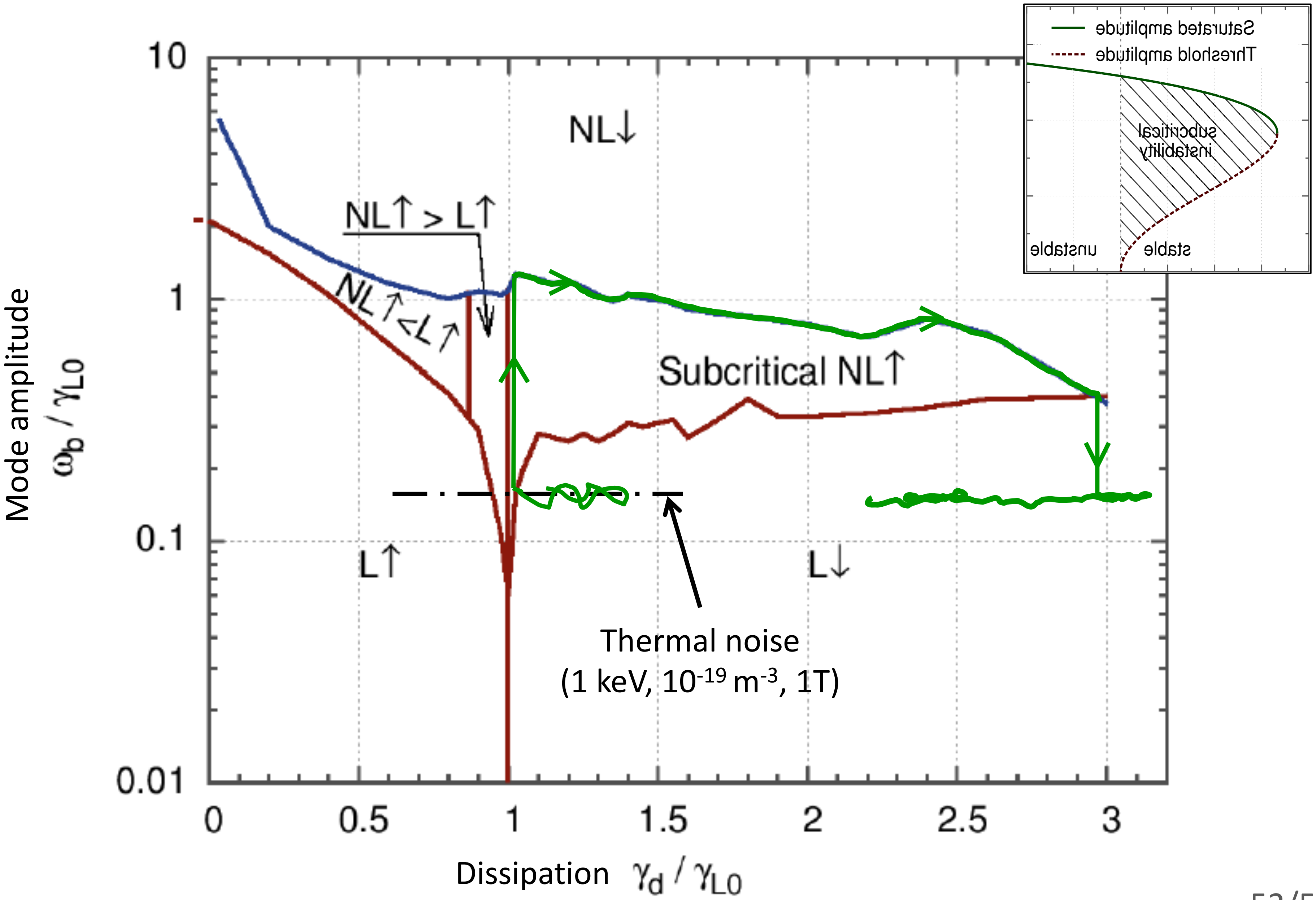
Stable, $\gamma < 0$

Nonlinearly unstable, $\gamma < 0$
(Subcritical instability)

Unstable, $\gamma > 0$



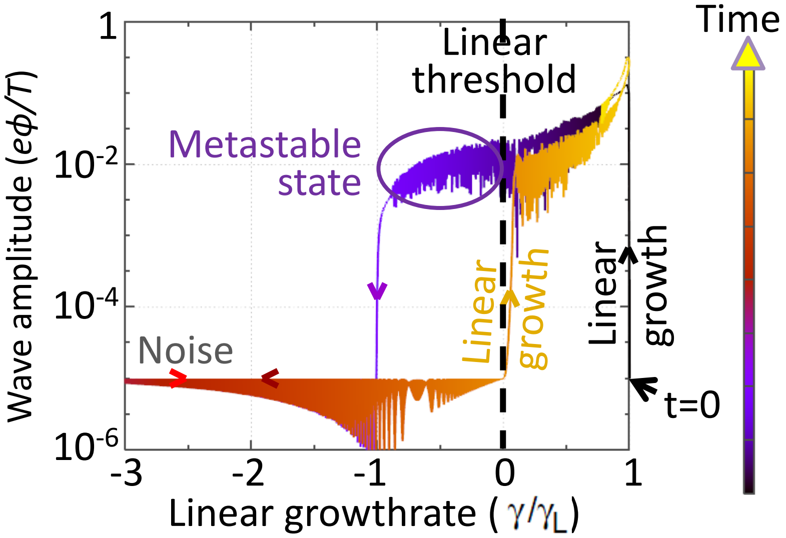
Nonlinear stability diagram of the BB model



Application: control in phase-space

Naive (linear) strategy

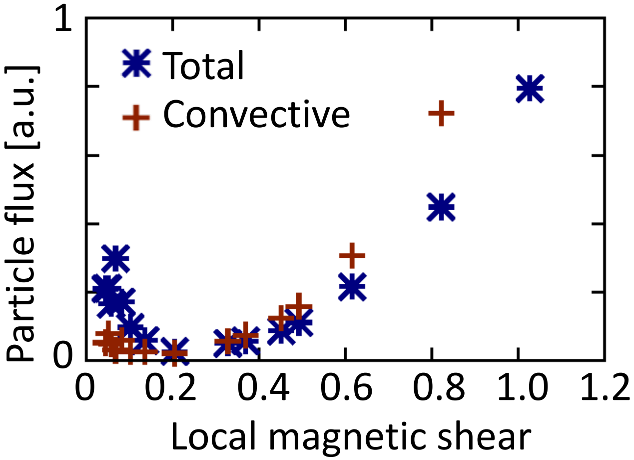
Decrease linear growthrate



The system self-organize to extract more free energy

Efficient (nonlinear) strategy

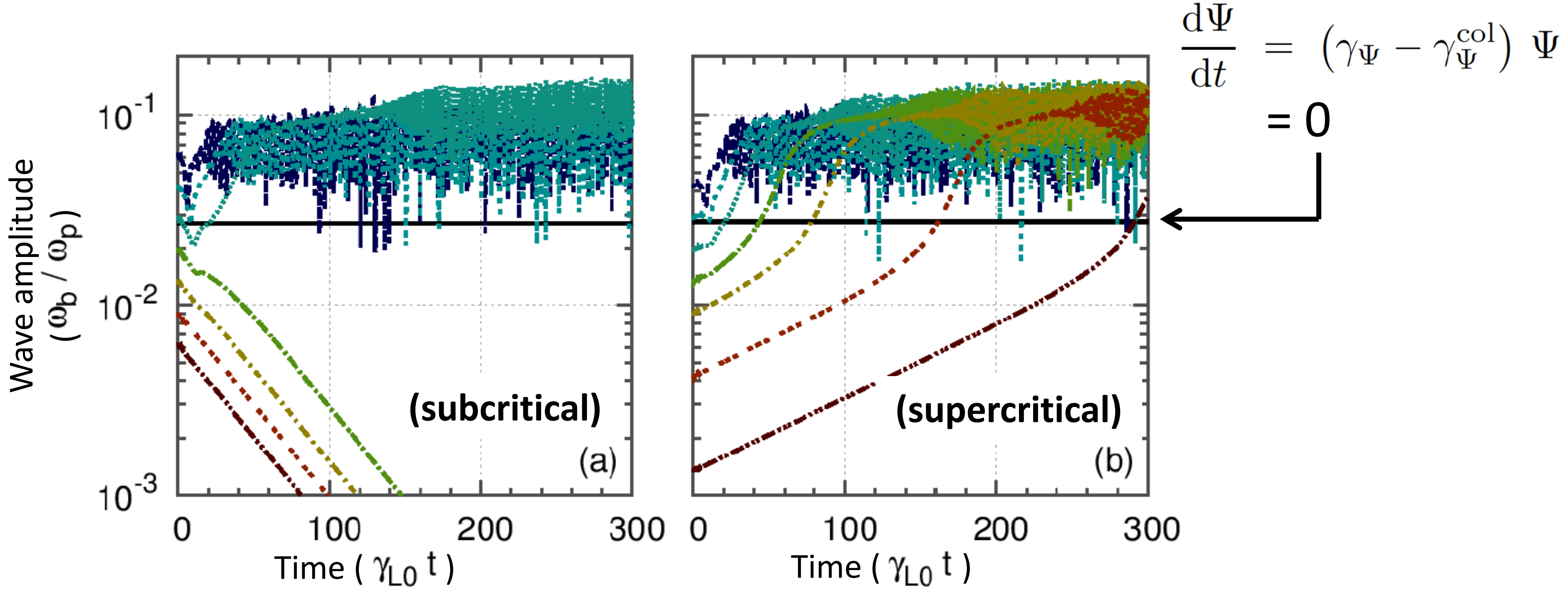
Diffuse PS structures, e.g. by modifying local magnetic shear



Bifurcation towards a quieter quasilinear regime

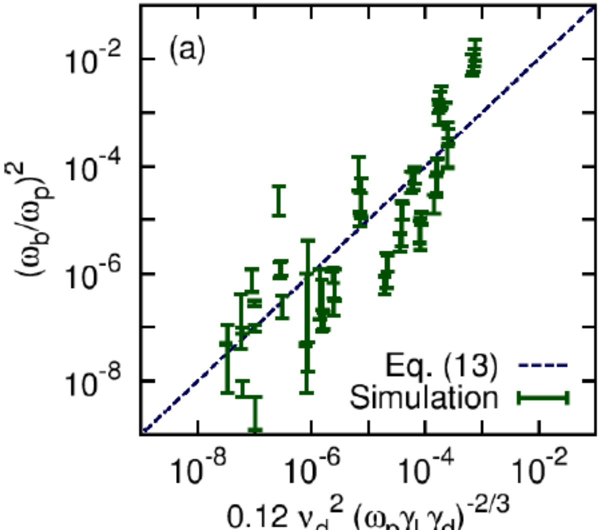
⇒ **Towards mitigation technique?**

Nonlinear amplitude threshold



$$\frac{d\Psi}{dt} = \left(\gamma_\Psi - \gamma_\Psi^{col}\right) \Psi$$

= 0



Lesur, Diamond '13

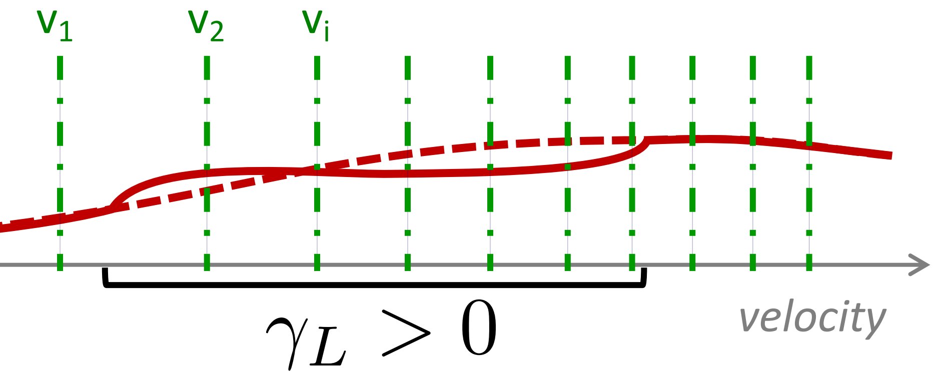
⇒ Experimental signatures of nonlinear growth: growthrate increases with amplitude, and amplitude threshold

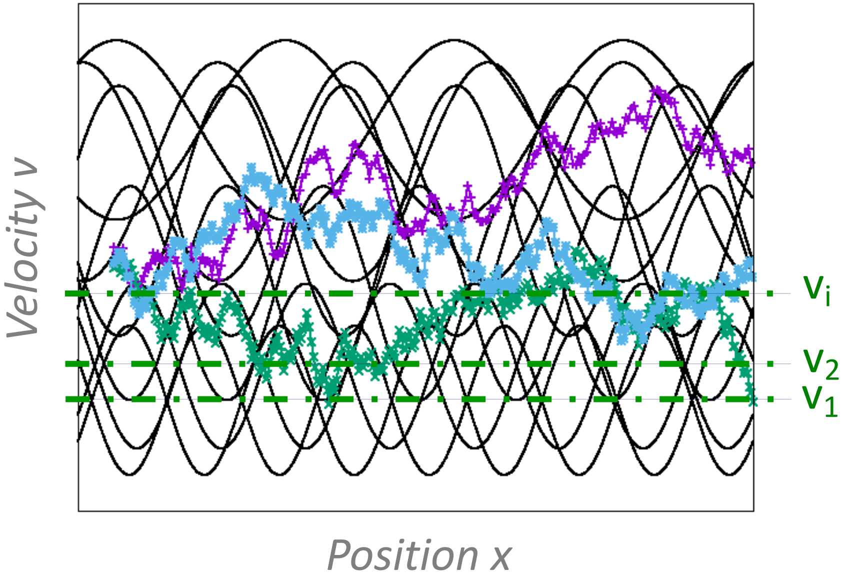
Outline

I. Bump-on-tail instability
II. The Berk-Breizman model
III. Chirping
IV. Subcritical instabilities

➡ Perspectives

Many-modes BoT instability





Quasilinear theory

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v}\left(D_{\mathrm{QL}}\frac{\partial \tilde{f}}{\partial v}\right)$$

$$D_{\mathrm{QL}} \sim \sum_k |E_k|^2/k$$

$$\frac{\partial |E_k|}{\partial t} = \gamma_L |E_k|$$

Flattening in the region $\gamma_L > 0$

Vedenov, Velikov, Sagdeev '61

Drummond & Pines '62

Sagdeev & Galeev '69

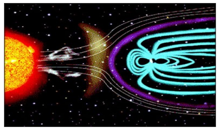
Outside the scope of QL theory:

- Phase-space turbulence, granulation
- Subcritical transport
- Strong fluctuations *Guillevic '23*

⇒ Many open questions remain

More general roles of phase-space structures

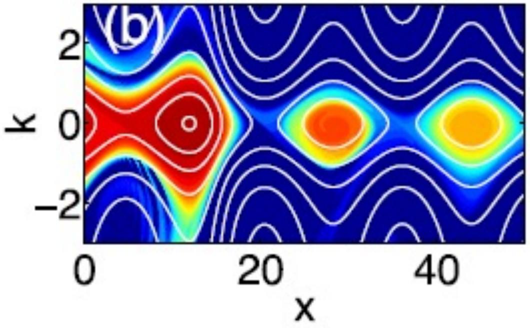
Vortices in phase-space are observed in experiments



- Space plasmas *Review: Eliasson & Shukla ‘06*
- Laboratory linear plasmas *Saeki ‘79*
- Magnetic reconnection of toroidal plasmas *Fox ‘08*
- Fusion plasmas *Kusama ‘99 ; Berk, Breizman & Pekker ‘96*
- Laser plasmas *Sarri ‘10*

Deep implications for instabilities, turbulence, transport, heating

- Drive nonlinear instabilities *Dupree ‘82*
- Modify the magnitude of saturation, spectrum of turbulence *Terry ‘90*
- Qualitative effect on transport *Biglari ‘88*
- Interact with large-scale flows *Kosuga ‘11*
- Propagation of trapped turbulence *Sasaki ‘17*



Analogy with descriptions of turbulence in real space

Collection of waves

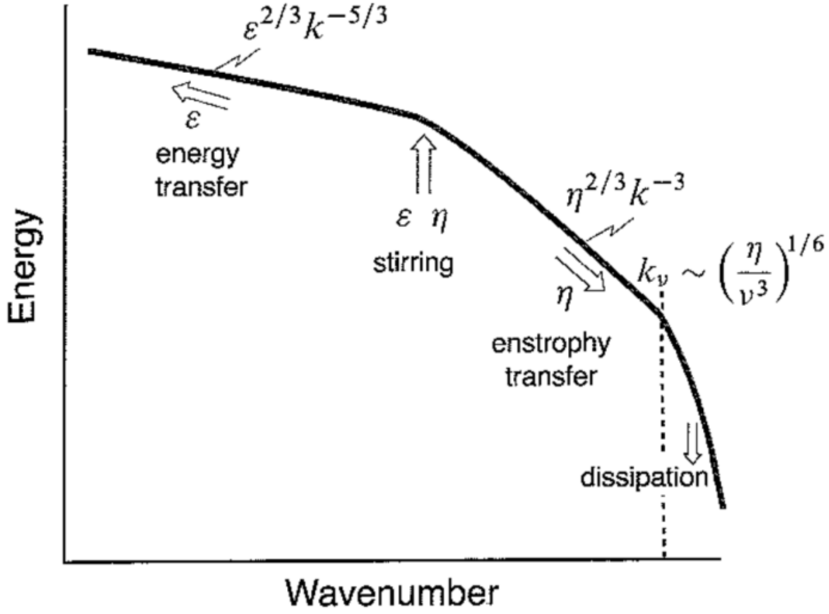
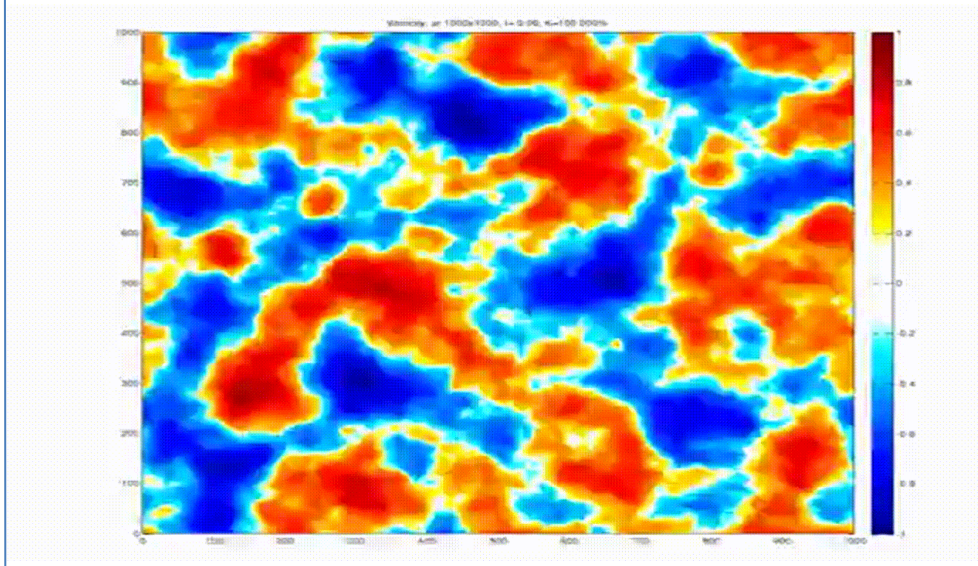


Illustration from Vallis '10

- Energy transfers?
- Phase dynamics?
- Intermittency?

$\Rightarrow$ Limited range of applicability

Collection of vortices



Vorticity in 2D Euler turbulence

$\Rightarrow$ **Reduction of dimensionality**

- Reduced model for smaller scales?

Backup slides follow

Difficulty of predicting&measuring linear rates

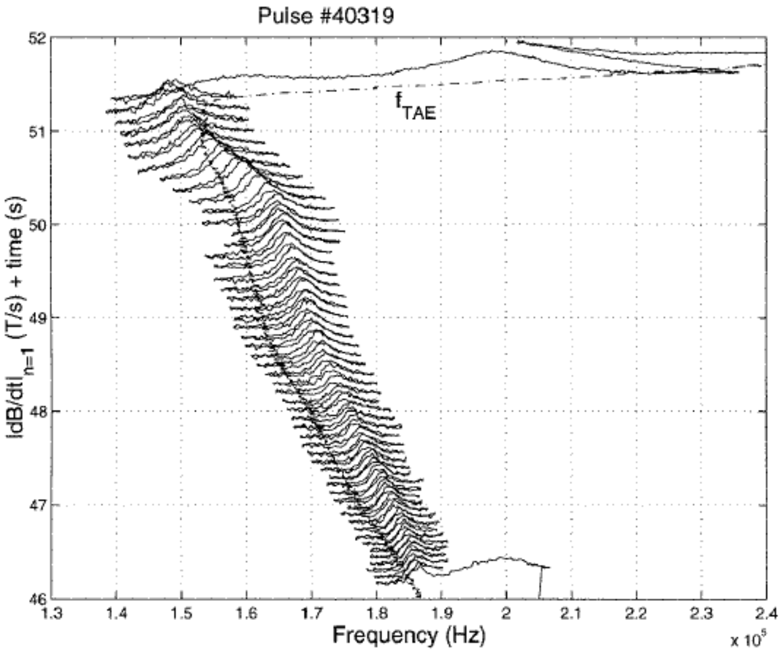
- γ_L depends on: - the gradients of the distribution function in energy and in P_ζ.
  - the alignment of the orbit with the eigenmode,
  - the strength of the various resonances,

  Too complicated to calculate analytically in general.

  Numerically, requires kinetic-MHD computations and internal diagnostics.

- γ_d involves still-debated complex mechanisms:
  - Ion Landau damping *Betti, Freidberg, PF(74)*
  - Radiative damping *Mett, Mahajan, PF(92)*
  - Collisional damping by trapped electrons *Gorelenkov, Sharapov, Phys.Scr.(92)*
  - Continuum damping *Rosenbluth, et al., PRL(92)* *Zonca, Chen, PRL(92)*

  Measured in dedicated experiments only.



Fasoli, et al., PPCF(97)

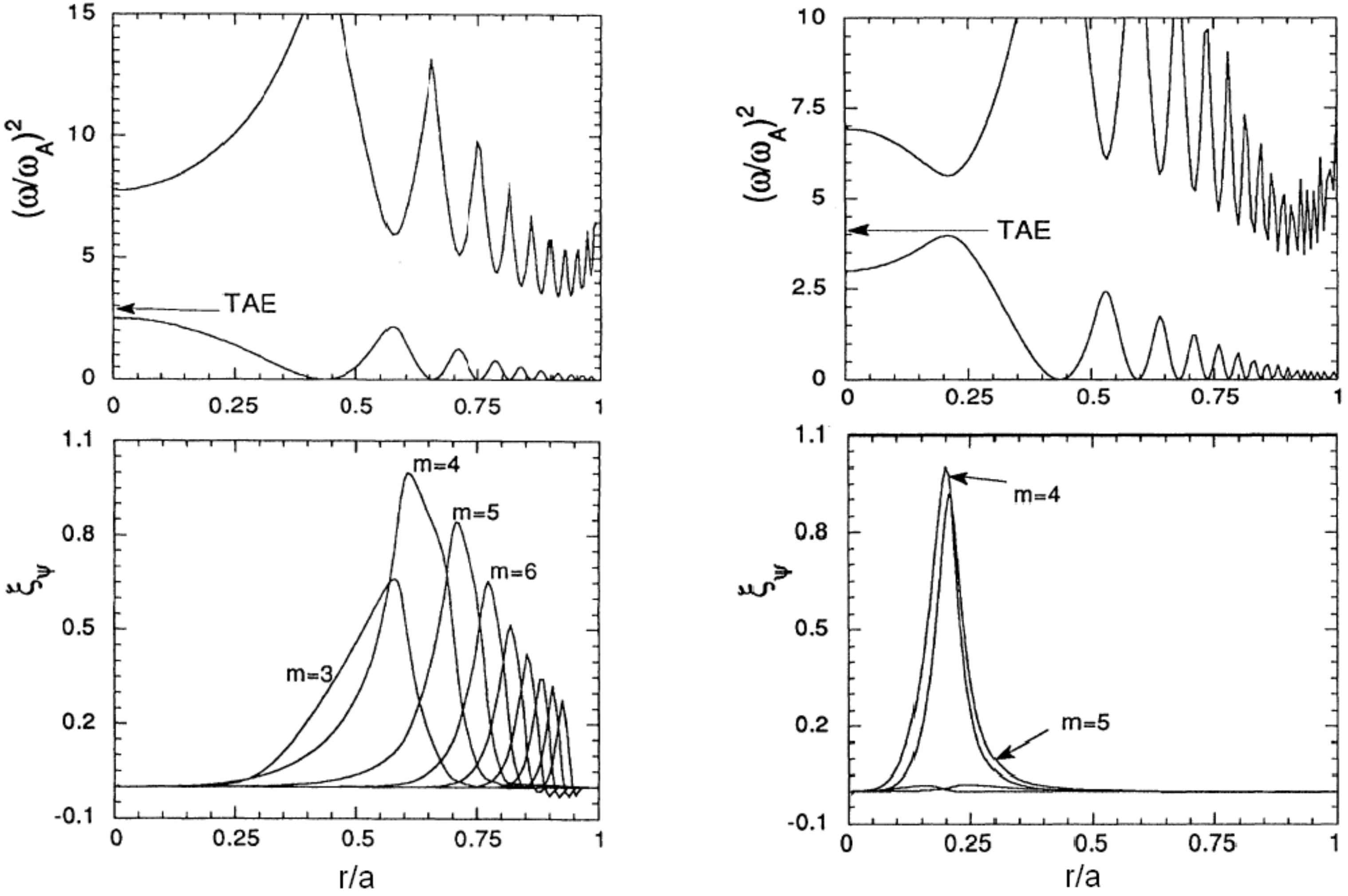
TAE structure

Typical TAE

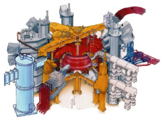
Core-localized TAE

SAW Continuum

MHD Displacement



Fu, et al., PRL(95)



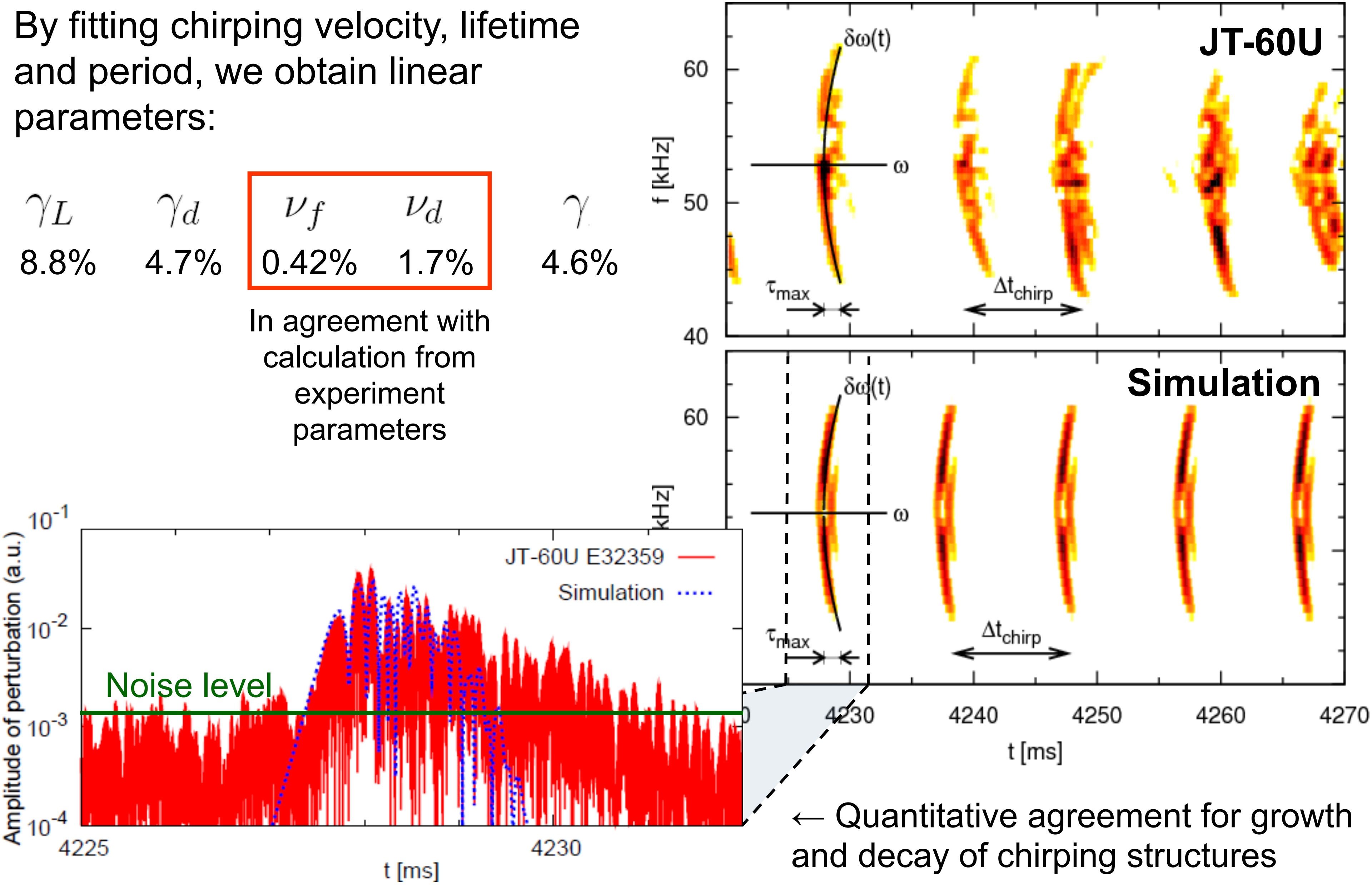
Analysis of JT-60U

Lesur, et al.
PoP(10)

By fitting chirping velocity, lifetime and period, we obtain linear parameters:

γ_L	γ_d	ν_f	ν_d	γ
8.8%	4.7%	0.42%	1.7%	4.6%

In agreement with calculation from experiment parameters

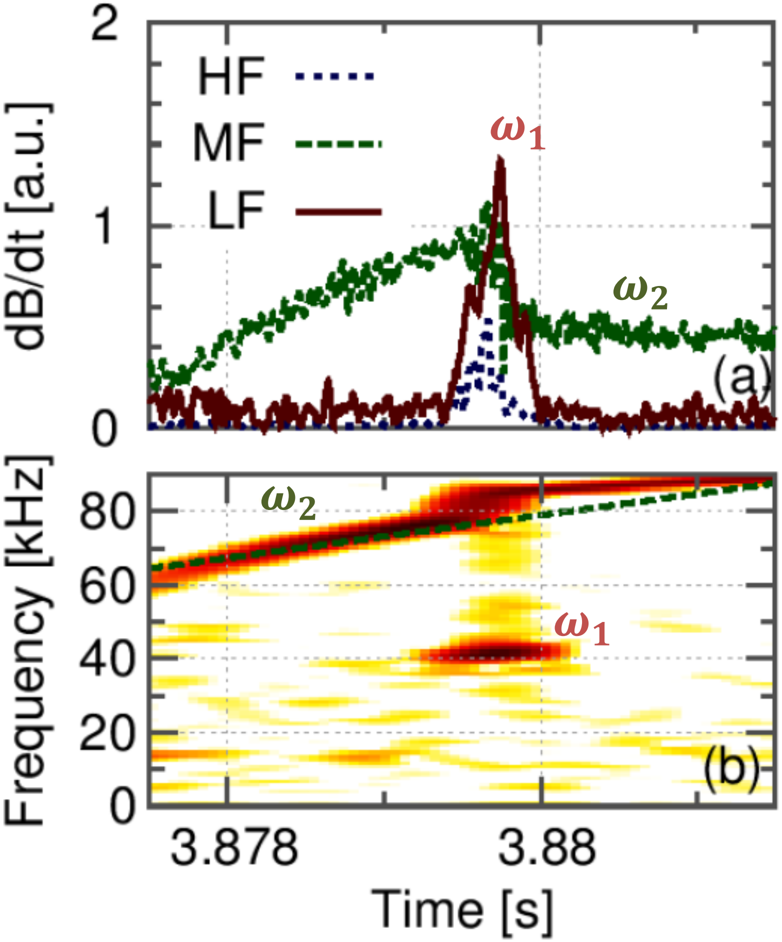


← Quantitative agreement for growth and decay of chirping structures

New model can reproduce the observation

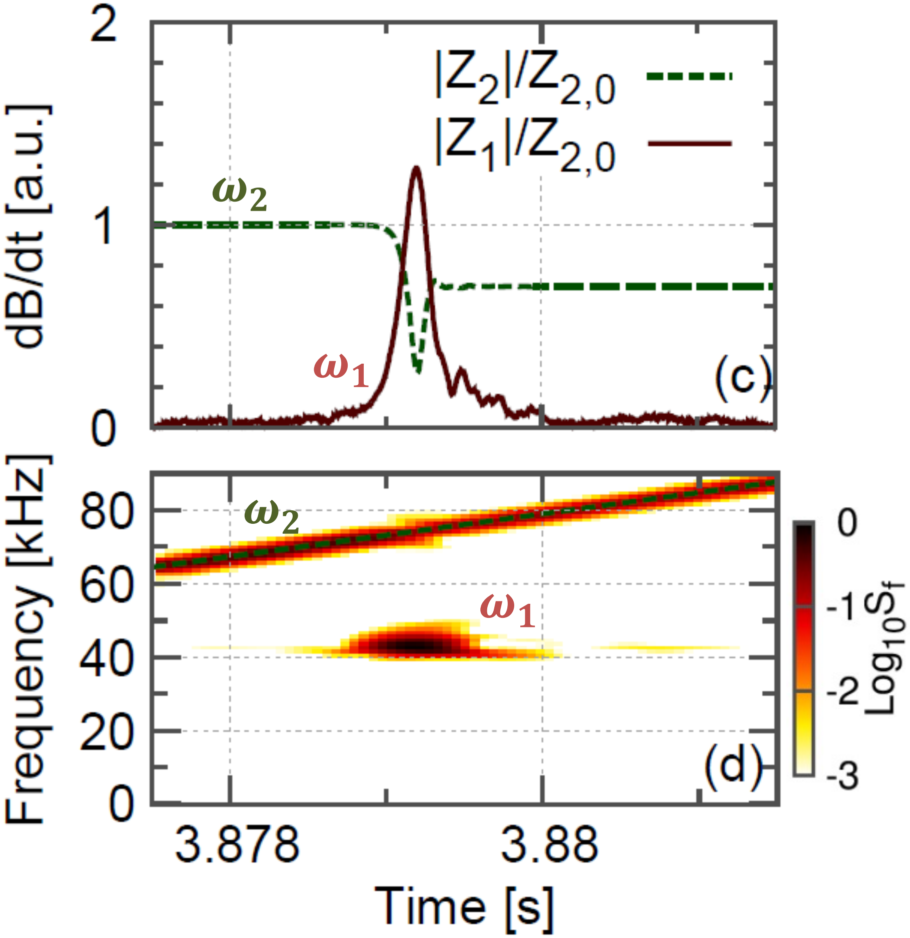
Experiment

Ido ’16

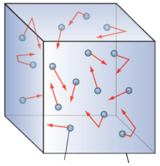


Simulation

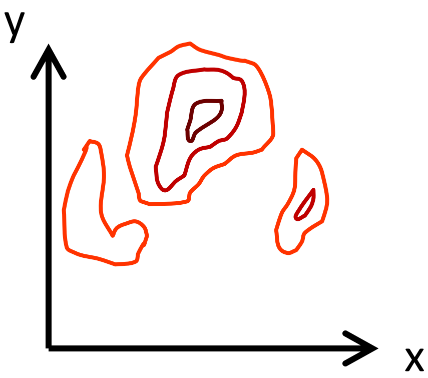
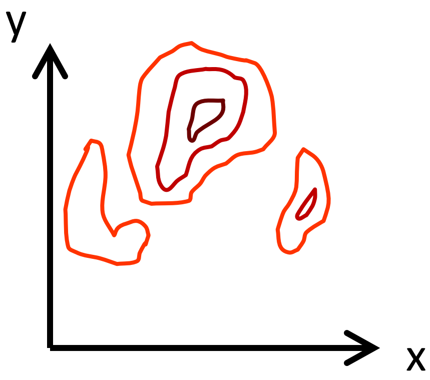
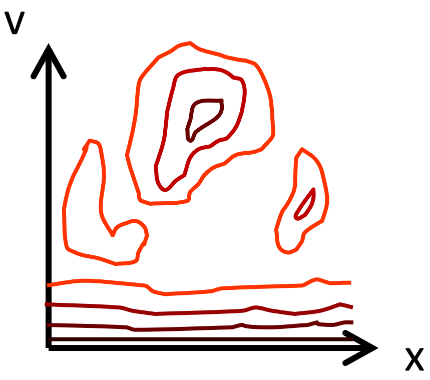
Lesur ’16



Our new model, which couples 1D kinetic equation with wave coupling equations, reproduces many features of the experiment.



Kinetic theory

System	**Many incompress. fluids in 2D**	**Quasi-geostrophic fluid**	**1D collisionless plasma**
Distribution	$n(x,y,t)$	$\omega(x,y,t)$	$f(x,v,t)$
Description space	2D config. space 	2D config. space 	2D phase space 
Continuity equation	$\frac{\partial n}{\partial t}+u_x\frac{\partial n}{\partial x}+u_y\frac{\partial n}{\partial y}=0$	$\frac{\partial \omega}{\partial t}+u_x\frac{\partial \omega}{\partial x}+u_y\frac{\partial \omega}{\partial y}=0$	$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}+E\frac{\partial f}{\partial v}=0$
Self-consistency	Hierarchy of fluid equations + closure	Stream function $\omega=\nabla^2\psi$	Poisson $\frac{\partial E}{\partial x}=\sum_s q_s\int f_s\,dx$